CONTROLLING REGIONAL MONOPOLIES IN THE NATURAL GAS INDUSTRY: THE ROLE OF TRANSPORT CAPACITY

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Controlling regional monopolies in the natural gas industry: The role of transport capacity†

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Abstract
This paper analyzes some optimal fiscal, pricing, and capacity investment policies for controlling regional monopoly power in the natural gas industry. By letting the set of control instruments available to the social planner vary, we provide a characterization of the technological and demand conditions under which “excess” capacity in the transport network arises in response to the loss of the two other control instruments, namely, transfers and pricing. Hence, the analysis yields some insights on an economy’s incentives to invest in infrastructures for the purpose of integrating geographically isolated markets.

JEL-code: L51, L95
Key words: Market power, Natural gas, Excess transport capacity.

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†Corresponding author: J.D. Oviedo, Universidad del Rosario, Bogota, Colombia, juan.oviedo@urosario.edu.co. This work has tremendously benefited from joint work and discussions with Jean-Jacques Laffont who died on May 1, 2004 leaving us deep sorrow. Any weakness of this work is however only ours.
1 Introduction

Following the US and the UK that reformed their natural gas industries in the late 70s and the 80s respectively, the EU has launched in the late 90s structural policies for enhancing gas-to-gas competition with the objective of complete liberalization of the market by 2007. More recently, EU Member States have been heavily investing in the development of their pipeline networks and Liquefied Natural Gas (LNG) liners. Such investments can be seen as driven by the need to anticipate growth of demand and import dependency. Indeed, gas penetration in energy consumption across activities in Europe has increased from less than 10% in the 70s to a current level of about 25% with an external dependency around 50%. Still, some observers have come to wonder whether such large-scale investments in capacity expansion are all that needed (Junola, 2003) and can be justified only on demand pressure and security of supply grounds. In this paper, we focus on the industrial organization role of transport capacity investments, namely, on their impact on market structure and the exercise of market power in the natural gas industry.

Since bringing the benefits of competition to consumers is a stated goal of the EU gas directive adopted in 1998 and amended in 2003 and given the high concentration of both commodity supply and transport in the EU region, it makes sense to investigate the role of network investments in the liberalization process. An issue that is particularly important for the EU is the nature of policies that should accompany this liberalization process and their effectiveness in mitigating the economic distortions that would result from a competitive market structure which is expected to be at best imperfect in the foreseeable future. A widely used transitory instrument for fostering gas-to-gas competition that has had varying degrees of success throughout the countries of the Union is gas release. This paper considers fiscal, pricing, and investment in capacity policies for improving market

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1 Algeria, Norway, and Russia are the main suppliers for Europe.

2 Chaton et al. (2008) have analyzed gas release as a short term instrument with the objective of fostering gas-gas competition and Chaton et al. (2012) have examined its interaction with investments in transport capacity.
efficiency and focuses on the role of transport network expansion.³

The relationship between network capacity and market structure in energy markets has attracted the attention of both empirical and theoretical economists. For the case of the US gas industry, a large empirical literature has examined the impact of investments in sub-network interconnection on the degree of market integration and the level of competition (see, among others, Doane and Spulber, 1994, and De Vany and Walls, 1994). From a more theoretical perspective, in electricity, a stream of the literature has examined the direct impact of transmission capacity on local market power (see, e.g., Borenstein et al., 2000 and Léautier, 2001) reaching the conclusion that transmission link expansion is effective for promoting competition. Building on a framework developed in Cremer et al. (2003), Cremer and Laffont (2002) argue that countering local market power in the natural gas industry might necessitate building excess transport capacity. McAfee and Reny (2007) also highlight the role of excess transport capacity in the determination of market power in natural gas transportation markets. The purpose of this paper is to further investigate the relationship between network size and regional market power.

At this initial stage of our investigation, the theoretical framework on which our analysis rests assumes away information problems.⁴ We consider a social planner who possesses complete information on demand and technology and controls the market power of an incumbent monopoly in a regional commodity gas market with a set of three instruments: transfers between consumers and the firm, pricing of the gas commodity, and investments in the capacity of the transport network. Starting from a situation where this whole set of control instruments is available to the social planner and reducing this set by first removing transfers and then transfers and price, should make the social planner rely more intensively on transport capacity.

³ Note that the problem of market power due to geographic isolation, which is the subject of this paper, is common in economics. Breaking up the isolation by investing in means of communications is obviously a solution but how much to invest is the relevant question.

⁴ Gasmi and Oviedo (2010) use a similar complete information framework to show how regulation of the upstream transport activity interacts in a socially optimal way with downstream gas-gas imperfect competition.
in order to counter monopoly power. Hence, fulfilling this objective without the ability to use transfers and control price can be expected to require a strictly higher level of transport capacity. Giving a characterization of the technological and demand conditions under which this “excess capacity hypothesis” holds is the main motivation of this paper. Our analytical strategy consists first in characterizing the optimal policies under the alternative control schemes, i.e., the optimal levels of the instruments used in each control regime, and then examining the relative levels of capacity these policies prescribe. These comparisons allow us to assess the extent to which transport capacity compensates for the lack/loss of the two other instruments of market power control. This analysis of optimal dimensioning of networks thus yields some insights on society’s incentives to invest in infrastructure in increasingly liberalized markets.

The plan of the paper is as follows. The next section describes the model of the industry configuration we consider and its basic theoretical ingredients. Sections 3, 4, and 5 characterize the optimal policies under three control schemes. These schemes are, respectively, one that lets the social planner have the largest set of instruments of market power control, namely, transfers, price, and capacity, one in which transfers are not allowed, and one in which the social planner controls only the capacity of the transport network. Section 6 focuses on the capacity variable and presents results on the excess capacity hypothesis. We summarize our main findings, discuss some of their policy implications, and give some directions for further research in the conclusion. Formal proofs and some illustrations of the optimal policies using some specific functional forms for the demand and cost functions are given in the appendix.

2 Industry configuration

Consider a regional natural gas commodity market, market $M$, supplied by a monopoly, firm $m$, producing with a technology described by a cost function $C_m(q_m) = \theta q_m + F_m$, where $q_m$ is output, $\theta$ is marginal cost, and
$F_m > 0$ is fixed cost.\footnote{We assume that the fixed cost $F_m$ is bounded and later provide a technical justification for this assumption. Even though shutting down the firm is sometimes prescribed by the optimal policies considered in this paper, the financing of this fixed cost is always taken into account.} Gas is also supplied at marginal cost $c$ in a competitive market, market $C_p$, which is geographically distinct from market $M$ but could be linked to it if a pipeline of capacity $K$ is built at cost $C(K)$, where $C(\cdot)$ is increasing convex, $C'(0) = 0$, and $C''(0) > 0$. Figure 1 below gives a schematic representation of this industry configuration. We assume that the regional monopoly’s marginal cost is greater than marginal cost of gas produced in market $C_p$, i.e., $\theta > c$.\footnote{This assumption reflects the standard productive inefficiency consequence of market power.} Gas produced under competitive conditions in market $C_p$ and imported into the regional commodity gas market $M$ should counter the exercise of market power by firm $m$ in this market.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{industry_configuration.png}
\caption{Industry configuration}
\end{figure}

Our analysis rests on the presumption that the very reason for a social planner to support a policy of building a transport line that links these two markets is to allow imports of gas from market $C_p$ into market $M$ that would bring consumers in this latter market the benefits of competition. Letting $Q_M(\cdot)$ represent these consumers’ demand function, assumed to be downward-sloping and concave, if a quantity of gas corresponding to full capacity of the pipeline $K$ is shipped from the competitive market into the regional market, firm $m$ remains a monopoly on the residual demand $Q_M(p_M) - K$, where $p_M$ is price. We assume that the social planner knows the demand and cost functions $Q_M(\cdot)$ and $C_m(\cdot)$ and seeks to determine policies that would restrain the firm from exerting its monopoly power in the regional market. An obvious yet important policy would be to interconnect this regional market and the competitive market $C_p$ with a pipeline. The question then is what the optimal size of this pipeline should be and the answer to this question should clearly depend on what other instruments the social planner has to mitigate the monopoly power of firm $m$. 

As is common in public economics, we assume that this public intervention takes place under second-best conditions in which public funds are raised through distortionary taxes at a (social) cost of $\lambda > 0$. Also, in the industry configuration considered in the paper we focus only on demand in the regional market and any pricing policy implemented in this market wouldn’t affect welfare in the competitive market where price is at the first-best level (marginal cost $c$). Hence, without loss of generality, we do not include welfare achieved in this competitive market into the analysis.\(^7\)

We start from a situation where the social planner has the ability to control the regional monopoly by means of three instruments, namely, (possibly two-way) transfers between consumers and the firm, price, transport capacity of the network, and hence monopoly output. We then restrict the set of available control instruments. We first consider the case where the social planner may not use transfers when setting the price and capacity levels. Then, we examine the situation where in addition to the fact that transfers are not allowed, the social planner can only influence the gas commodity price in the regional market through transport capacity that affects the residual demand of the monopoly.

3 Controlling the regional monopoly with transfers, price, and transport capacity

In this section, we assume that the social planner may use public funds to make transfers between consumers and the firm. These funds are raised through taxation that generates welfare losses so that a monetary transfer to the firm $T$ costs society $(1 + \lambda)T$ where $\lambda$ is the cost of public funds. Let $S(\cdot)$ represent gross surplus of consumers in market $M$. Total supply of gas $Q_M(p_M)$ in this market, composed of $K$ units imported from the competitive market and $q_m$ units produced locally by firm $m$, brings taxpayers an

\(^7\)Another factor that is excluded from the analysis without affecting the main qualitative results is the marginal cost of transport which here is normalized to zero. Alternatively, one can assume that this marginal cost of transport is a positive constant and include it in $c$, i.e., write $c = c_p + c_t$ where $c_p$ is now the marginal cost of production in the competitive market and $c_t$ is the marginal cost of transport.
aggregate (net) welfare \( V \) given by

\[
V = \{S(Q_M(p_M)) - p_MQ_M(p_M)\} + \{(1 + \lambda)(p_M - c)K - C(K)\} - \{(1 + \lambda)T\}
\]  

This taxpayers’ welfare comprises three parts: the net surplus of consumers in the regional market \( M \), the social valuation of profits generated by the \( K \) units of gas imported from the competitive market, and the social cost of the transfer \( T \) made to the firm. The welfare of firm \( m \) is measured by its utility \( U \) that sums its profits from sales and the transfer it receives:

\[
U = \{(p_M - \theta)[Q_M(p_M) - K] - F_m\} + T
\]  

When controlling the regional monopoly, the social planner has to account for the participation constraint of the firm and the constraint of non-negativity of its output: \(^8\)

\[
U \geq 0 \quad q_m = Q_M(p_M) - K \geq 0
\]  

The utilitarian social welfare function \( W \) is the sum of taxpayers’ welfare \( V \) and firm’s utility \( U \). Substituting for \( V \) from (1) and for \( T \) from (2) yields social welfare

\[
W = \{S(Q_M(p_M)) + \lambda p_MQ_M(p_M)
- (1 + \lambda)[\theta(Q_M(p_M) - K) + cK + C(K) + F_m]\} - \lambda U
\]  

as the social valuation of total production minus its social cost, minus the social opportunity cost of the firm’s utility. From this expression of social welfare we see that reducing the monopoly’s utility, its “rent,” is socially desirable for, as can be seen from (2), this utility includes a transfer of public funds collected through distortive taxation. Similarly, we see from (5) that the social valuation of total production explicitly includes the fiscal value of the revenues that it generates \( \lambda p_MQ_M(p_M) \). \(^9\)

\(^8\)The output nonnegativity constraint needs to be taken into account here because transfers \( T \) (here unconstrained in sign and magnitude) can be used to finance any fixed cost that wouldn’t be recovered through revenues from gas.

\(^9\)Indeed, these revenues allow the government to rely less on public funds raised through taxation at a deadweight loss.
With transfers, monopoly output, and capacity as instruments of control, the social planner’s program consists in maximizing social welfare \( W \) given by (5) with respect to \( p_M, K, \) and \( U \), under the firm’s participation and output nonnegativity constraints, respectively (3) and (4).\(^{10}\) Letting \( \phi \) and \( \nu \) denote the Lagrange multipliers associated with these two constraints respectively, and using the fact that \( \partial S(Q_M) / \partial Q_M = p_M \), the following first-order conditions obtain:\(^{11}\)

\[
\begin{align*}
\lambda Q_M + (1 + \lambda) (p_M - \theta) Q'_M + \nu Q'_M &= 0 \\
(1 + \lambda) [(\theta - c) - C'(K)] - \nu &= 0 \\
-(\lambda - \phi) &= 0 \\
\phi U &= 0 \\
\nu [Q_M - K] &= 0
\end{align*}
\]

From (8) and (9), we immediately see that the participation constraint is binding, i.e., \( U = 0 \) and, indeed, transfers allow the social planner to totally extract (finance) the firm’s profit (deficit). Letting \( \varepsilon(Q_M) \) designate the price-elasticity of demand in market \( M \), the first-order conditions (6)-(10) allow us to state the following proposition:\(^{12}\)

**Proposition 1** When price (or equivalently output) and capacity are both controlled by the social planner and, in addition, the latter can use public funds to make transfers between consumers and the firm, one of two following policies \((K, p_M, \nu)\) arises:

(i) The policy \((0 < K < Q_M, p_M > \theta, \nu = 0)\) in which the local monopoly meets part of the market demand and price and capacity satisfy

\[
\frac{p_M - \theta}{p_M} = \frac{p_M - (c + C'(K))}{p_M} = \frac{\lambda}{1 + \lambda \varepsilon(Q_M)}
\]

\[
(1 + \lambda)C'(K) = (1 + \lambda)(\theta - c)
\]

\(^{10}\)Note that as long as the social planner controls monopoly output and transport capacity, he totally controls price in market \( M \).

\(^{11}\)To minimize notation and where it doesn’t lead to any ambiguity, the arguments of some of the demand and cost functions will be dropped in the presentation.

\(^{12}\)An illustration of the approach used to solve the social planner program is provided in the appendix. The proof of Proposition 1 and those of the other propositions in this paper are also given in the appendix.
(ii) The policy \((K = Q_M, p_M > c, \nu > 0)\) in which the local monopoly is shut down, market demand is entirely met through imports, and the markup of the import activity is given by

\[
\frac{p_M - (c + C'(Q_M))}{p_M} = \frac{\lambda}{1 + \lambda \varepsilon(Q_M)}
\]

(13)

Under policy (i), the condition \((\theta - c) < C'(Q_M)\) holds, i.e., the firm’s marginal cost, \(\theta\), is smaller than the marginal cost of imports when the latter meet the entire market demand, \(c + C'(Q_M)\). Under policy (ii) the reverse is true.

Note that, thanks to the availability of transfers, the policies described in Proposition 1 are not responsive to the value of the fixed cost, \(F_m\). Under both policies we see from equations (11) and (13) that pricing obeys a Ramsey principle according to which the price markup is inversely proportional to the price-elasticity of demand in the regional market.\(^\text{13}\) When \(\nu = 0\), i.e., when the local monopoly is active, it is indeed optimal to let it apply a markup (see (11)) since public funds are costly and the social planner can use transfers to capture this markup. As to capacity, it is set such that the social marginal cost of imports, \((1 + \lambda)(c + C'(K))\), is equal to the social marginal cost of local production, \((1 + \lambda)\theta\), a relationship that can be seen from (12). When \(\nu > 0\), i.e., when the firm is shut down, its fixed cost is financed through transfers and there is still a markup but now the relevant marginal cost is that of imported gas (see (13)).

4 Controlling the regional monopoly with price and capacity only

We now assume that the social planner can still set the transport capacity and the firm’s output level, and hence fully controls price in market \(M\), but transfers between consumers and the firm are no longer permitted. Social

\(^{13}\) Note that here the coefficient of proportionality is a function of \(\lambda\) and hence, in contrast to the standard Ramsey formulas, is exogenous.
welfare $W$ is expressed as

$$W = \{S(Q_M(p_M)) - p_M Q_M(p_M)\}$$

$$+ \{(1 + \lambda) [(p_M - c)K - C(K)]\}$$

$$+ \{(p_M - \theta)(Q_M(p_M) - K) - F_m\}$$

(14)

that is, as the sum of the net consumer surplus, the social value of the profits generated by the $K$ units imported from the competitive market, and the profits of the firm that now cannot be transferred to consumers. Gathering terms, we obtain

$$W = S(Q_M(p_M)) + \lambda p_M K$$

$$- [\theta(Q_M(p_M) - K) + F_m] - (1 + \lambda) [cK + C(K)]$$

(15)

Cross-examining (5) and (15), we see that as now transfers are not allowed, the social planner assigns a fiscal value $\lambda p_M K$ only to the revenues generated by the $K$ units shipped from the competitive market $C_p$ into the regional market $M$.

The social planner maximizes social welfare given by (15) with respect to price and capacity, under the participation constraint (nonnegativity of profits), that now does not include transfers, and the firm’s output nonnegativity constraint

$$\Pi_m = (p_M - \theta)(Q_M(p_M) - K) - F_m \geq 0$$

$$q_m = Q_M(p_M) - K \geq 0$$

(16)

(17)

Given that transfers are not allowed, it makes sense for us to consider only the policies with $p_M > \theta$ when the firm is active, which is always the case from (16) since we assume $F_m > 0$.\(^{14}\) Hence, from now on, we focus on the set defined by the participation constraint (16) hereafter referred to as the participation set.

Letting $\phi$ denote the Lagrange multiplier associated with the participation constraint, the system of first-order conditions that characterize the

\(^{14}\)When there is no fixed cost, i.e., $F_m = 0$, cases where the firm is shut down may arise. In such cases, because the social planner does not face the concern of financing a fixed cost, the relevant constraint is (17).
optimal policy is given by

$$\lambda K + (p_M - \theta)Q'_M + \phi [(p_M - \theta)Q'_M + (Q_M - K)] = 0$$  \hspace{1cm} (18)$$

$$(1 + \lambda) [(\theta - c) - C'(K)] + (\lambda - \phi) (p_M - \theta) = 0$$ \hspace{1cm} (19)$$

$$\phi [(p_M - \theta) (Q_M - K) - F_m] = 0$$ \hspace{1cm} (20)$$

$$(p_M - \theta) (Q_M - K) - F_m \geq 0$$ \hspace{1cm} (21)$$

To rule out the possibility of having $K < 0$ we assume that the fixed cost is bounded so that

$$F_m \leq -\frac{\lambda Q_M^2 + \left[(1 + \lambda)(\theta - c)Q'_M + \sqrt{\Gamma}\right]}{2(1 + \lambda)Q'_M} Q_M$$ \hspace{1cm} (22)$$

where $\Gamma \equiv [\lambda Q_M + (1 + \lambda)(\theta - c)Q'_M]^2 - 4(1 + \lambda)^2(\theta - c)Q_MQ'_M$.\textsuperscript{15}

The next proposition characterizes the alternative pricing and transport capacity policies associated with this two-instrument control scheme.

**Proposition 2** When price (or equivalently output) and capacity are both controlled by the social planner but the latter cannot use public funds to make transfers between consumers and the firm, there are two exclusive candidate policies $(K, p_M, \phi)$ with one of them having two possible forms:

\textsuperscript{15}This upper-bound is obtained as follows. When $F_m > 0$, we look for the conditions characterizing a policy of the type $(0, p_M > \theta, \phi > 0)$. Substituting for $K = 0$ in the system of first-order conditions (18)-(20), we see that such a policy is defined by

$$\phi = \lambda + \frac{(1 + \lambda)(\theta - c)Q_M}{F_m} \text{ and } \phi Q_M + \frac{(1 + \phi)F_m Q_M}{Q_M} = 0.$$ 

Solving for $F_m$, yields the right-hand side term of the inequality (22). It is easy to see that any $F_m$ smaller than this term will indeed yield $K > 0$. Moreover, it can be shown that (22) implies $F_m \leq -Q_M^2/Q'_M$, which is a condition that ensures that the participation set be nonempty for nonnegative values of $K$. The latter condition is derived as follows. For the participation set to be nonempty for $K \geq 0$ it suffices that the largest $K$ that makes the participation constraint binding be nonnegative. Such a $K$ is found by solving the following program:

$$\max_{p_M, K} K \text{ s.t. } (p_M - \theta) [Q_M(p_M) - K] - F_m = 0 \quad K \geq 0$$

It is then easy to show that such a capacity level satisfies $[Q_M F_m + (Q_M - K)^2]/F_m \leq 0$. Now, if this inequality holds for $K = 0$, it will clearly hold for any $K > 0$.\textsuperscript{11}
(i) The policy \((K = 0, p_M > \theta, \phi > \lambda)\) which consists in building no capacity and letting the local monopoly earn a markup that makes it just break even:

\[
\frac{p_M - \theta}{p_M} = \frac{\phi}{1 + \phi \varepsilon(Q_M)}
\]

(ii) The policy \((0 < K < Q_M, p_M > \theta, \phi \geq 0)\) which prescribes building capacity and setting price above marginal cost. This policy takes one of the two following forms:

(a) The policy \((0 < K < Q_M, p_M > \theta, \phi = 0)\), characterized by

\[
\frac{p_M - \theta}{p_M} = \frac{\lambda K}{Q_M \varepsilon(Q_M)} \frac{1}{1 + \lambda Q_M \varepsilon(Q_M)}
\]

\[
\frac{p_M - (c + C'(K))}{p_M} = \frac{\lambda K}{1 + \lambda Q_M \varepsilon(Q_M)} \frac{1}{1 + \lambda Q_M \varepsilon(Q_M)}
\]

\[
(1 + \lambda)C'(K) = (1 + \lambda)(\theta - c) - \lambda^2 \frac{K}{Q_M}
\]

(b) The policy \((0 < K < Q_M, p_M > \theta, \phi > 0)\) in which the pricing and capacity building rule are characterized by

\[
\frac{p_M - \theta}{p_M} = \left[ \frac{\lambda K + \phi(Q_M - K)}{(1 + \phi)Q_M} \right] \frac{1}{\varepsilon(Q_M)}
\]

\[
\frac{p_M - (c + C'(K))}{p_M} = \left[ \frac{\lambda K + \phi(Q_M - K)}{(1 + \lambda)Q_M} \right] \frac{1}{\varepsilon(Q_M)}
\]

\[
(1 + \lambda)C'(K) = (1 + \lambda)(\theta - c) + \frac{\Upsilon}{Q_M - K}
\]

where \(\Upsilon \equiv m_{im} \times [\lambda Q_M(Q_M - K) + (1 + \lambda)Q'_M F_m] > 0\), and

\[
m_{im} = \frac{p_M - \theta}{(Q_M - K) + (p_M - \theta)Q'_M} = \frac{F_m}{Q'_M F_m + (Q_M - K)}
\]

Under policy (i) the fixed cost hits its bound, i.e., the condition (22) is satisfied with equality. Whenever (22) is satisfied with strict inequality only policies of type (ii) may arise. Policy (ii-a) corresponds to the case where the marginal cost of the regional monopoly, \(\theta\), equals the “net” social marginal cost of imports, \((1 + \lambda)[c + C'(K)] - \lambda p_M\), i.e., \(Q'_M \left[ C''(K) - \frac{C'(K)}{K} \right] - \frac{\lambda K}{Q_M} Q''_M C''(K) < 0\), and the firm’s variable profits are larger than the fixed
cost, i.e., \( F_m < -\lambda K \frac{(Q_M - K)}{Q_M} \). Policy (ii-b) corresponds to the case where either of these inequalities is reversed.

From Proposition 2, we see that when the solution of the constrained welfare maximization program allows the monopoly to earn positive profits (case where \( \phi = 0 \)), i.e., under policy (ii-a), it makes a markup which is inversely related to the elasticity of demand and increases with the share of imports in the total consumption of gas in the regional market. The reason for this latter result is that the social marginal valuation of capacity increases with price. As to the markup made on imports, it is increasing with the share of these imports in total demand but it is less sensitive to it than the firm’s markup. From the capacity building rule under this policy, we see that the social cost of the marginal unit of gas shipped from the competitive market, \((1 + \lambda) [c + C'(K)]\), net of the fiscal revenue of this imported gas unit, \(\lambda p_M\), equals the social cost of having this unit produced by the local monopoly, \(\theta\).

When it is optimal to let the local monopoly active and just break even ((\(q_m > 0\), \(K > 0\), and \(\phi > 0\))), i.e., under policy (ii-b), the markup made by the monopoly is again inversely related to the regional market demand elasticity. However, the proportionality term is the ratio of the fiscal valuation of the revenues from imports, \(\lambda p_M K\), plus the valuation the planner assigns to the fact that revenues made by the firm help to relax the participation constraint, \(\phi p_M q_m\), to the social valuation of the aggregate revenues in the regional market in the case where these revenues were exclusively generated by the firm, \((1 + \phi)p_M Q_M\). The markup from imports has a similar structure but the denominator of the proportionality term is the social valuation of aggregate revenues in the case where total demand is met by imports, \((1 + \lambda)p_M Q_M\).\(^{16}\) Under this zero-profit policy, optimal capacity makes the “net” social cost of the marginal unit of gas shipped from the competitive market, \((1 + \lambda) [c + C'(K)] - \lambda p_M\), just equal to the social cost of having this unit produced by the local monopoly, \(\theta\), net of the value the planner assigns to the contribution of this unit to the relaxation of the firm’s participation constraint.

\(^{16}\)This interpretation is obtained after multiplying the right-hand side expressions of (27) and (28) by \(p_M/p_M\).
constraint, $\phi(p_M - \theta)$. Indeed, these profits can no longer be collected by the planner who now lacks the instrument (transfers) that would allow him to do so.

Finally, under policy (i) the fixed cost is so high that it is optimal to let the regional monopoly cover full market demand and hence no capacity is built ($K = 0$).

5 Controlling the regional monopoly with transport capacity only

We now assume that the social planner lacks an additional instrument of control, namely, setting the monopoly’s level of output, and hence he can only partially affect price in market $M$ through the residual demand. Transport capacity is therefore the only instrument left to him to counter the exercise of local market power by the firm in this market. In practice though, we model this case as if the social planner continues to set the price level, but now this price has to fall within a profit-maximizing-constrained set of values. Let us be more specific.

For a given volume of gas $K$ imported from the competitive market, the firm remains a monopoly in its local commodity gas market on the residual demand $Q_M(p_M) - K$. Given this demand, the firm sets price so as to maximize its profit $\Pi_m$ given by

$$\Pi_m = (p_M - \theta)[Q_M(p_M) - K] - F_m$$

The first-order condition of this profit-maximization problem is

$$\frac{\partial \Pi_m}{\partial p_M} = Q_M'(p_M) - \theta = 0$$

while the second-order condition that ensures that we are indeed at a maximum is $\Omega \equiv (p_M - \theta)Q_M'' + 2Q_M' < 0$.

Given that transfers are not allowed, the form of the social welfare function for this control scheme is analogous to the one described in the previous
section which we restate here:

\[ W = S(Q_M(p_M)) + \lambda p_M K - \theta(Q_M(p_M) - K) - (1 + \lambda) [cK + C(K)] - F_m \] 

(32)

The program of the social planner consists in maximizing social welfare given by (32) with respect to \( p_M \) and \( K \), under the regional monopoly participation constraint, \( \Pi_m \geq 0 \), where \( \Pi_m \) is given by (30), its output nonnegativity constraint, \( q_m \geq 0 \), and its profit-maximization constraint (31).\(^{17}\) As in the previous section, let us focus on policies with \( p_M > \theta \) in which case the firm’s output nonnegativity constraint can be ignored.\(^{18}\) Letting \( \phi \) and \( \eta \) designate the Lagrange multipliers associated with the firm’s participation and profit-maximization constraints, respectively, we obtain the following first-order conditions:\(^{19}\)

\[
\begin{align*}
\lambda K + (p_M - \theta) Q'_M - \eta \Omega &= 0 \\
(\lambda - \phi) (p_M - \theta) + (1 + \lambda) [(\theta - c) - C'(K)] + \eta &= 0 \\
\phi [(p_M - \theta)(Q_M - K) - F_m] &= 0 \\
(p_M - \theta)(Q_M - K) - F_m &\geq 0 \\
(p_M - \theta)Q'_M + Q_M - K &= 0
\end{align*}
\]

(33) \quad (34) \quad (35) \quad (36) \quad (37)

The following proposition summarizes the policies characterized by these first-order conditions.

**Proposition 3** \(^{15} \) When capacity is the only instrument controlled by the social planner, it is always built and there are two exclusive candidate policies \((K, p_M, \phi, \eta)\):

\(^{15}\)Strictly speaking, the second-order condition of the firm’s profit-maximization program should also be taken as a constraint. The standard way to deal with this issue, is to check ex post that this second-order condition is satisfied by the solution of the program. \(^{18}\) Indeed, \((p_M - \theta) > 0 \) and (31) imply \( q_m \geq 0 \). Note that in this case there is no need for a constraint on the size of the marginal cost gap. \(^{19}\)The Lagrange multiplier associated with the profit-maximization constraint (31), \( \eta \), is interpreted as the social marginal cost of letting the regional monopoly maximize profits. Indeed, \( \eta > 0 \) implies that a reduction in the optimal price markup made by the firm results in a higher level of welfare. However, note that from the cross-partial derivative of the welfare function (32), \( \partial^2 W / \partial p_M \partial K = \lambda \), a reduction in the price markup leads to a decrease in the optimal capacity level. In particular, when \( \eta > 0 \) a reduction in import capacity is welfare improving.
(i) The policy \(0 < K < Q_M, p_M > \theta, \phi = 0, \eta \neq 0\) characterized by

\[
\frac{p_M - \theta}{p_M} = \left[\frac{\lambda K - \eta \Omega}{Q_M}\right] \frac{1}{\varepsilon(Q_M)} = \left[\frac{Q_M - K}{Q_M}\right] \frac{1}{\varepsilon(Q_M)} \quad (38)
\]

\[
\frac{p_M - (c + C'(K))}{p_M} = \left[\frac{\lambda K - \eta (\Omega - Q'_M)}{(1 + \lambda)Q_M}\right] \frac{1}{\varepsilon(Q_M)} \quad (39)
\]

\[(1 + \lambda)C'(K) = (1 + \lambda)(\theta - c) - \frac{\lambda(Q_M - K)}{Q'_M} - \frac{Q_M - (1 + \lambda)K}{\Omega} \quad (40)
\]

where \(\Omega\) is as defined above.

(ii) The policy \(0 < K < Q_M, p_M > \theta, \phi > 0, \eta \neq 0\) characterized by the average cost pricing rule

\[
\frac{p_M - \theta}{p_M} = \left[\frac{Q_M - K}{Q_M}\right] \frac{1}{\varepsilon(Q_M)} = \sqrt{-Q'_M F_m} \frac{1}{\varepsilon(Q_M)} \quad (41)
\]

\[
\frac{p_M - (c + C'(K))}{p_M} = \left[\frac{(1 + \phi)[\lambda K - \eta \Omega + \eta Q'_M]}{(1 + \lambda)Q_M}\right] \frac{1}{\varepsilon(Q_M)} \quad (42)
\]

\[K = Q_M - \sqrt{-Q'_M F_m} \quad (43)
\]

Under policy (i) the marginal cost of the local monopoly \(\theta\) plus the shadow cost of the firm’s profit maximization constraint, \(\eta\), equals the “net” social marginal cost of imports, \((1 + \lambda)[c + C'(K)] - \lambda p_M\), and the resulting firm’s variable profits are larger than the fixed cost, i.e., \(F_m < \frac{(Q_M - K)^2}{Q'_M}\). Under policy (ii) this condition holds with equality, i.e., \(F_m = -\frac{(Q_M - K)^2}{Q'_M}\).

Proposition 3 says that when the solution allows for positive profits by monopoly (\(\phi = 0\)), i.e., under policy (i), this firm earns a markup which is proportional to the share of its output in the aggregate demand and inversely related to the elasticity of demand. The size of this markup is larger (smaller) than that under the control scheme where the social planner had total control over pricing, described in section 4, if the shadow cost of the firm’s profit maximization constraint, \(\eta\), is positive (negative). Optimal capacity is determined by balancing the net social cost of having an extra unit imported from the competitive market, \((1+\lambda)[c+C'(K)] - \lambda p_M\), against
the cost of having that unit produced locally by the monopoly, $\theta$, plus the social cost of complying with the profit-maximization constraint, $\eta$.

When this mono-instrument control scheme yields no profits for the local monopoly at the optimum ($\phi > 0$), i.e., under policy (ii), the firm’s markup is inversely related to the elasticity of demand but increases with the size of the fixed cost. Since in this particular case capacity is used by the social planner as a residual instrument to make the firm just break even, it is decreasing in $F_m$ (see (43)).

6 Role of transport capacity

So far, we have characterized optimal policies obtained under three control schemes that are differentiated by the set of control instruments available to the social planner. More specifically, we have considered the benchmark case in which the social planner can use transfers, capacity, and price to mitigate regional monopoly power. Then, we have studied the more realistic cases in which first, transfers are not allowed, second, neither transfers nor price control are possible. By comparing the levels of transport capacity achieved under these three alternative market power control schemes, we now provide a characterization of some conditions that would be interpreted as leading to “excess” capacity in the sense that, under these conditions, optimal policies command systematically higher levels of transport capacity.

For clarity of exposition, we refer to the schemes described in section 3 (control of price and capacity with transfers), 4 (control of price and capacity without transfers), and 5 (control of capacity only) as schemes $A$, $B$, and $C$ respectively. We study the evolution of network capacity as the set of instruments that the planner uses maximize social welfare gets reduced. Letting $K^A$, $K^B$, $K^C$, and $p^A_M$, $p^B_M$, $p^C_M$ designate the optimal levels of transport capacity and price achieved under the respective control schemes, we proceed by pairwise comparisons and identify some conditions under which the excess capacity hypothesis holds, i.e., under which the loss/lack of control instruments necessitates higher investments in transport capacity.
in order to mitigate regional monopoly power.

6.1 Excess capacity due to the loss of transfers as a control instrument

When analyzing the impact of the loss of (only) the ability to use transfers between consumers and the firm, the relevant comparison is between schemes A and B. We express the first-order conditions of the constrained welfare maximization programs under these schemes, (6), (7), and (18), (19) as follows:

\[
\frac{\partial W^A}{\partial p_M} + \nu^A Q'_M = 0 \tag{44}
\]

\[
\frac{\partial W^A}{\partial K} - \nu^A = 0 \tag{45}
\]

\[
\frac{\partial W^B}{\partial p_M} + \phi^B \frac{\partial \Pi_m}{\partial p_M} = 0 \tag{46}
\]

\[
\frac{\partial W^B}{\partial K} - \phi^B (p_M - \theta) = 0 \tag{47}
\]

Examining the left-hand sides of (44), (46), and (45), (47), we see that

\[
\frac{\partial W^B}{\partial p_M} = \frac{\partial W^A}{\partial p_M} - \lambda \frac{\partial \Pi_m}{\partial p_M} \tag{48}
\]

\[
\frac{\partial W^B}{\partial K} = \frac{\partial W^A}{\partial K} + \lambda (p_M - \theta) \tag{49}
\]

A casual look at (44)-(49) suggests that the (endogenous) shadow cost of the constraint of nonnegativity of the firm’s output, \( \nu^A \), the (endogenous) shadow cost of its participation constraint, \( \phi^B \), and the (exogenous) social cost of public funds, \( \lambda \), are going to influence the relative optimal levels of transport capacity. The following proposition formalizes this relationship.

**Proposition 4** The loss of (only) transfers as a control instrument has the following consequences. When the shadow cost of the participation constraint under scheme B, \( \phi^B \), is smaller than the social cost of public funds, \( \lambda \),

---

\(^{20}\)Each pairwise comparison is illustrated by using specific functional forms and particular parameter values. This empirical analysis is based on simulations with respect to two sensitivity parameters that play an important role in our theoretical model, namely, the marginal cost gap \((\theta - c)\) and firm \( m \)'s fixed cost \( F_m \).
i.e., \((\lambda - \phi^B) > 0\), society suffers a net marginal cost from letting the firm make positive profits under this scheme and “excess” capacity is needed, i.e., \(K^B > K^A\).

As an illustration of this proposition, Figures 2a and 2b below exhibit the sign of the capacity differential, \((K^B - K^A)\), and that of \(\nu^A\) and \(\phi^B\), in terms of the marginal cost gap, \((\theta - c)\), and the fixed cost, \(F_m\), assuming the functional forms given by

\[
Q_M(p_M) = \gamma - p_M, \quad C(K) = \frac{\omega}{2}K^2; \quad \gamma, \omega > 0, \quad \gamma > c
\]  

and for the grid of parameters values \((\lambda, \omega, \gamma, c) \in \{(1/3, 1/2, 10, 2), (1/3, 1/15, 10, 2)\}\).\(^{21}\)

\(^{21}\)These two sets of parameter values allow us to examine both the case where the polynomial \(\omega(1 + \lambda) - \lambda^2\) is positive and negative.
Cross-examining the upper and lower parts of Figures 2a and 2b, we see that whenever the solution under scheme $B$ is interior ($\phi^B = 0$), so is the solution under $A$ ($\nu^A = 0$), and $K^B > K^A$. Note that, as stated in the proposition, these figures show that the sign of the capacity differential $(K^B - K^A)$ is the same as that of $(\lambda - \phi^B)$. 

Figure 2a: $(K^B - K^A)$, $\nu^A$, $\phi^B$ with $\omega(1 + \lambda) - \lambda^2 > 0$

Figure 2b: $(K^B - K^A)$, $\nu^A$, $\phi^B$ with $\omega(1 + \lambda) - \lambda^2 < 0$
6.2 Excess capacity due to the loss of price control

Suppose now that the social planner initially has two control instruments, price and capacity, and then loses the ability to set price. In order to analyze the impact of such a reduction in the set of control instruments, the relevant comparison is between schemes B and C. Since the welfare functions of these two schemes are identical and scheme C has an additional constraint (the firm’s profit-maximization constraint), let us express its first-order conditions (33), (34), and (37) as

\[
\frac{\partial W_B}{\partial p_M} - \eta^C \Omega = 0 \quad (51)
\]

\[
\frac{\partial W_B}{\partial K} - \phi^C (p_M - \theta) + \eta^C = 0 \quad (52)
\]

\[
\frac{\partial \Pi_m}{\partial p_M} = 0 \quad (53)
\]

These first-order conditions give us reasons to expect that the shadow costs of the participation constraint under B and C, \(\phi^B\) and \(\phi^C\), and that of the profit-maximization constraint under C, \(\eta^C\), are going to be influential in the determination of the relative size of transport capacity. These expectations are confirmed in the next proposition.

**Proposition 5** When price and capacity are first controlled by the social planner and then the latter loses price control, the impact on network capacity is as follows. When the social marginal cost of letting the firm maximize profits is positive, i.e., when \(\eta^C > 0\), the lose of price control by the social planner entails “excess” capacity, i.e., \(K^C > K^B\).

Figures 3a and 3b below show the sign of the capacity differential, \((K^C - K^B)\), and that of \(\phi^B\), \(\phi^C\), and \(\eta^C\) in terms of the marginal cost gap, \((\theta - c)\), and the fixed cost, \(F_m\), assuming the functional forms given by (50) and under for the grid of parameter values used in the previous subsection.
Comparing the upper and lower parts of Figures 3a and 3b, we see that whenever the solution under scheme $C$ yields $\eta^C > 0$, the capacity differential is such that $K^C - K^B > 0$. With the functional forms (50), condition $Q'_M \left[ C''(K) - \frac{C'(K)}{K} \right] - \frac{\lambda K}{Q'_M} Q''_M C''(K) < 0$ does not hold and Figure 3b confirms the statement in Proposition 2 that whenever $\omega(1 + \lambda) - \lambda^2 < 0$, the solution under $B$ has $\phi^B > 0$.\(^{22}\)

\(^{22}\)This condition represents the inequality $(1 + \lambda)Q'_M C''(0) + \lambda^2 > 0$ stated in Proposition 2.
6.3 Excess capacity due to the loss of both transfers and price as control instruments

Finally, let us assume that the social planner initially has three control instruments, price, capacity, and transfers, and then can neither use transfers nor set price. The effect of such a removal of two control instruments can be analyzed by comparing schemes $A$ and $C$. Let us express the first-order conditions associated with scheme $C$, (33), (34), and (37) as

\[
\frac{\partial W^A}{\partial p_M} - \lambda \frac{\partial \Pi_m}{\partial p_M} - \eta C \Omega = 0 \quad (54)
\]

\[
\frac{\partial W^A}{\partial K} + (\lambda - \phi C)(p_M - \theta) + \eta C = 0 \quad (55)
\]

We see from these first-order conditions that the shadow cost of the constraint of nonnegativity of the firm’s output under $A$, $\nu^A$, that of the participation constraint under $C$, $\phi^C$, and that of the profit-maximization constraint under $C$, $\eta^C$, should play an important role in the determination of the relative size of transport capacity. The next proposition clarifies this role.

**Proposition 6** When price, capacity, and transfers are initially available to the social planner as tools to mitigate monopoly power and then he loses the ability to use transfers and set price, then, provided that after the reduction in the set of control instruments the firm earns strictly positive profits, when the social marginal cost of letting the firm maximize profits is positive, i.e., when $\eta^C > 0$, the loss of the two control instruments entails “excess” capacity, i.e., $K^C > K^A$.

Figures 4a and 4b below show the sign of the capacity differential, $(K^C - K^A)$, and that of $\nu^A$, $\phi^C$, and $\eta^C$ in terms of the marginal cost gap, $(\theta - c)$, and the fixed cost, $F_m$, assuming the functional forms given by (50) and for the grid of parameter values previously used.
As stated in Proposition 6, we see from these figures that when firm’s profits are not only maximized ($\eta^C \neq 0$) but also strictly positive ($\phi^C = 0$), $\text{sign}[K^C - K^A] = \text{sign}[\eta^C]$ when $\eta^C > 0$, and hence $K^C > K^A$.23

23Observe from these figures that there does not exist a case where $\nu^A > 0$, $\phi^C = 0$, and $\eta^C > 0$. The reason for this is that if under $A$ the firm is shut down ($q_m = 0$) and if allowing it to maximize profits under $C$ is socially costly ($\eta^C > 0$), then there is no reason for letting it earn strictly positive profits under this scheme ($\phi^C > 0$).
7 Conclusion

The gas industry throughout the world, in particular in the European Union, has been facing an important question which is raised in most of the infrastructure sectors. In a context where reforms aimed at opening to competition some segments of the industry are conducted, how to make sure that the exercise of monopoly power by incumbents inherited from the historical market structure is not going to be an impediment to the liberalization process. This paper has provided an analysis of some policies that a social planner can use to mitigate regional monopoly power in the gas commodity market. We have considered optimal policies implementable through three control instruments, transfers, price, and transport capacity, and we have focused on the role of capacity.

As a starting point, we have considered a benchmark situation where the social planner, having complete information, may use transfers between consumers and a regional monopoly, control the gas commodity price, and set the capacity of a pipeline used to import competitive gas into the regional market. We then have examined the effect on the pipeline capacity of the planner’s loss of the ability to use transfers and control the price. A comparative analysis of these control mechanisms has allowed us to shed some light on the extent to which these various tools of mitigating regional market power interact, in particular, to show that transport capacity might be a good substitute to other “less controllable” instruments such as transfers and tariffs.

The analysis has also allowed us to explore the incentives of an economy to develop transport infrastructure in order to fight market power. In addition to the standard allocative inefficiency due to market power in a geographically isolated market, in our model with complete information the social planner has to account for a productive inefficiency and the financing of a fixed cost. Moreover, our model explicitly accounts for the fact that public funds are costly. Clearly then, the incentives of the social planner to build infrastructure capacity depend on many factors. These factors, as highlighted by our analysis, include the width of the panoply of control in-
struments that are available to the planner, how relatively inefficient the regional firm is, the magnitude of the burden imposed by the financing of the fixed cost, the cost structure of the capacity building activity, and how costly raising public funds through taxation is. Putting these factors together and solving the various tradeoffs involved is, as can be expected, not straightforward. Nonetheless, we derive some propositions that yield some instructive qualitative information on how the various control instruments interact and in particular the degree to which the social planner should intensify investments in infrastructure in order to exert competitive pressure on regional monopolies.

In the benchmark case where the social planner has full control of the regional firm through transfers, capacity, and price the only relevant factor is how severe the productive inefficiency is. If the marginal cost gap is substantially large, the social planner finds it worthwhile to intensively invest in transport capacity to the point of inducing the shutting down of the regional firm even if a fixed cost needs to be financed. If the marginal cost gap is small, the social planner finds it beneficial to put some, but not extreme, competitive pressure on the regional firm by moderately investing in transport capacity and letting the firm earn a markup that is recoverable through transfers anyway.

When transfers are no longer available but the social planner still controls capacity and price, it is optimal not to build capacity only in the case where the fixed cost is so large that the social planner is merely constrained to let the firm entirely meet market demand so as to earn enough profits to finance such an extremely large fixed cost. If the fixed cost is not prohibitively high, building capacity to generate some competition allowing both the firm and the import activity to earn markups is optimal.

When not only transfers but also pricing are beyond the social planner’s control, competitive pressure through investment in transport capacity, be it small, is always optimal. The extreme policy that consists in intensively investing in capacity to the point of inducing the shutting down of the regional firm is optimal only when the firm’s productive inefficiency is extremely high and the fixed cost to be financed is arbitrarily small.
Control of monopoly power is to a large extent the subject of regulatory economics. The purpose of this paper was to explore the analysis of the interaction among regulatory tools under the admittedly strong assumption of complete information. We have demonstrated that excess capacity can itself be considered as a strategic regulatory instrument. A necessary extension of our analysis is to introduce asymmetric information on the firm’s production technology. Our conjecture is that under incomplete information, the additional objective of controlling for the monopoly’s information rent will affect in some important ways the role of network capacity investments.
A Appendix

Controlling the regional monopoly with transfers, price, and transport capacity

Illustration of the program resolution approach: To study the solution to the system of first-order conditions (6)-(10), we proceed in two steps. First, we consider the unconstrained maximization program (maximization of (5)) in the capacity-price ($K - p_M$) space, and then we introduce the firm’s output nonnegativity constraint (4).24

An unconstrained welfare maximizing capacity-price pair satisfies the following first-order conditions25

$$
\lambda Q_M + (1 + \lambda)(p_M - \theta)Q'_M = 0 \quad (A.1)
$$

$$
(1 + \lambda)[(\theta - c) - C'(K)] = 0 \quad (A.2)
$$

For the social welfare function (5), $\text{sign}[\partial^2 W/\partial K\partial p_M] = 0$, which says that the social marginal valuation of capacity remains unaffected by changes in the regional market price.26 Hence, in the $K-p_M$ space, the first-order condition with respect to price (A.1) can be represented by a line parallel to the $K$-axis at the price level $p_M = \theta - \lambda Q_M/(1 + \lambda)$.27 Similarly, the first-order condition with respect to capacity (A.2) is a line parallel to the $p_M$-axis at the capacity level $K$ such that $(\theta - c) = C'(K)$, i.e., at $K = C'^{-1}(\theta - c)$.28 The unique solution to the system constituted of the two equations (A.1) and (A.2) corresponds then to the intersection of these two lines.

Next, the nonnegativity set defined by the constraint (4) has a boundary which is decreasing and concave with slope $1/Q'_M$ in the $K-p_M$ space. If the capacity-price pair that solves (A.1) and (A.2) yields $q_m > 0$, and this will be the case if and only if

$$
K = C'^{-1}(\theta - c) < Q_M(p_M)\big|_{p_M = \theta - \lambda Q_M/(1 + \lambda)Q'_M}, \quad (A.3)
$$

then this pair will also be the solution of the constrained program of the social planner. In this case, total demand in market $M$ cannot be met exclusively by imports $K$ at the prevailing price. Otherwise, the solution to the constrained maximization program will be at the tangency point of a welfare level curve and the boundary of the nonnegativity set characterized by:29

$$
-\frac{(1 + \lambda)[(\theta - c) - C'(Q_M)]}{\lambda Q_M + (1 + \lambda)(p_M - \theta)Q'_M} = \frac{1}{Q'_M} \quad (A.4)
$$

24Since $U = 0$, we can ignore the firm’s participation constraint (3).

25The welfare function given in (5) will be strictly concave if, for any capacity-price pair, the condition $(1 + \lambda)C''(K)[(1 + 2\lambda)Q'_M + (1 + \lambda)(p_M - \theta)Q''_M] < 0$ holds. As we assume both $C''(K) > 0$ for any $K \geq 0$ and a concave downward-sloping demand schedule, provided $(p_M - \theta) \geq 0$, the former condition is always satisfied. Thus, the optimal price and capacity levels are not only local but also global interior welfare maximizers.

26For a general convex firm’s cost function, $\text{sign}[\partial^2 W/\partial K\partial p_M] = \text{sign}[(1 + \lambda)C''(Q_M)]$ which is either negative or zero.

27Strict concavity of the social welfare function (5) insures that this differential equation defines a unique line for nonnegative prices.

28Note that since $C'$ is increasing convex, its inverse exists.

29Given our demand and capacity building cost assumptions, second-order conditions are always satisfied.
To further illustrate the resolution of this three-instrument control scheme, let us consider the case where demand is linear and the technology of capacity building is represented by a quadratic cost function. More specifically, let

\[ Q_M(p_M) = \gamma - p_M, \quad C(K) = \frac{\omega}{2} K^2; \quad \gamma, \omega > 0, \quad \gamma > c \quad (A.5) \]

With these functional forms, the first-order condition with respect to price (A.1) is a horizontal line crossing the \( p_M \)-axis at \( p_M = \theta + \frac{\lambda}{1 + 2\lambda} (\gamma - \theta) \), whereas that with respect to capacity (A.2) is a vertical line crossing the \( K \)-axis at \( K = (\theta - c)/\omega \). See Figures A1a and A1b. The shaded areas correspond to the set defined by the local monopoly output nonnegativity constraint (4) which here is the set of \((K, p_M)\) pairs satisfying \( K + p_M < \gamma \).

When (A.3) holds, we obtain the interior solution to (6)-(10) as the intersection of the two lines shown in Figure A1a. More specifically, when

\[ (\theta - c) < \left[ \frac{\omega(1 + \lambda)}{(1 + 2\lambda) + \omega(1 + \lambda)} \right] (\gamma - c) \quad (A.6) \]

the (interior) solution is

\[ K = \frac{(\theta - c)}{\omega} \quad (A.7) \]
\[ p_M = \theta + \left[ \frac{\lambda}{1 + 2\lambda} \right] (\gamma - \theta) \quad (A.8) \]

This solution corresponds to policy (i) of Proposition 1. When (A.3) does not hold, the boundary solution at the tangency point shown in Figure A1b is obtained. More specifically, when

\[ \left[ \frac{\omega(1 + \lambda)}{(1 + 2\lambda) + \omega(1 + \lambda)} \right] (\gamma - c) \leq (\theta - c) < (\gamma - c) \quad (A.9) \]

this boundary solution is

\[ K = \left[ \frac{1 + \lambda}{(1 + 2\lambda) + \omega(1 + \lambda)} \right] (\gamma - c) \quad (A.10) \]
\[ p_M = c + \left[ \frac{\lambda + \omega(1 + \lambda)}{(1 + 2\lambda) + \omega(1 + \lambda)} \right] (\gamma - c) \quad (A.11) \]
and corresponds to policy (ii) described in Proposition 1.

**Proof of Proposition 1:** With \((\theta - c) < C'(Q_M)\), the first-order condition (7) yields \(\nu = 0\) in which case (10) yields \(0 < K < Q_M\) and (6) and (7) are rewritten as (11) and (12). When \((\theta - c) \geq C'(Q_M)\), (7) yields \(\nu > 0\) in which case (10) yields \(K = Q_M\) and (6) combined with (7) yield (13).

Controlling the regional monopoly with price and transport capacity

**Illustration of the program resolution approach:** For the purpose of analyzing the solution to (18)-(21) in the \(K-p_M\) space, we first consider the unconstrained maximization program and then introduce the participation set. An unconstrained welfare maximizer capacity-price pair satisfies the following first-order conditions:

\[
\begin{align*}
\lambda K + (p_M - \theta) Q_M' &= 0 \\
(1 + \lambda) [(\theta - c) - C'(K)] + \lambda (p_M - \theta) &= 0
\end{align*}
\]  

(A.12) (A.13)

Second-order conditions for such an unconstrained local social welfare maximizer are synthesized by

\[
\frac{\lambda Q'_M}{\lambda K Q'_M - Q'_M^2} < \frac{(1 + \lambda)C''(K)}{\lambda}
\]  

(A.14)

Observe that, for the welfare function (15), \(\text{sign}[\partial^2 W/\partial K \partial p_M (= \lambda)] > 0\). Hence, under this control scheme without transfers, the social marginal valuation of capacity increases with the regional market price.

In the \(K-p_M\) space, provided that \(Q''_M \leq 0\) and \(C''(K) \geq 0\), the first-order condition with respect to price of the unconstrained program (A.12) can be represented by an increasing concave function, with slope \(\lambda Q'_M/[\lambda K Q'_M - Q'_M^2]\), which crosses the \(p_M\)-axis at \(p_M = \theta\). Similarly, the first-order condition with respect to capacity (A.13) can be represented by an increasing convex function, with slope \((1 + \lambda)C''(K)/\lambda\), which crosses the \(p_M\)-axis at \(p_M = \theta - ((1 + \lambda)(\theta - c)/\lambda) \leq \theta\). These two functions representing (A.12) and (A.13) cross at most twice for any \(K\) and at most once for \(K > 0\).

Since \((\theta - c) > 0\), at \(K = 0\) the increasing concave function representing (A.12) implies a strictly larger level of price than the one implied by the increasing convex function representing (A.13). Therefore, such functions are expected either to cross only once or not at all. It is straightforward to show that in the case they cross only once, the crossing point, which is a solution to (A.12)-(A.13), satisfies the second-order conditions (A.14) for the unconstrained welfare maximization program.

The participation set is a convex set in the \(K-p_M\) space when both \(q_m > 0\) and \(p_M > \theta\). Its boundary has a slope \(m_{\partial_m}\) given by

\[
m_{\partial_m} = \frac{F_m}{Q_m F_m + (Q_M - K)^2}
\]  

(A.15)

For a general convex cost function of the regional monopoly, \(\text{sign}[\partial^2 W/\partial K \partial p_M] = \text{sign}[\lambda + C''(Q_M)] \geq 0\). Therefore, in general the effect of an increase in the regional market price \(p_M\) on the social marginal valuation of capacity depends on the relative magnitude of \(\lambda\). This shows the simplification that the specific cost function \(C_m(\theta, q_m) = \theta q_m + F_m\) allows to achieve.
If the capacity-price pair that satisfies (A.12)-(A.14) belongs to the participation set, it will also be a solution to the constrained welfare maximization program. Otherwise, the constrained welfare maximizer is at a tangency point between a welfare level curve and the boundary of the participation set, characterized by:

\[ m_{m_t} = \frac{(1 + \lambda) \left[ (\theta - c) - C'(K) \right] + \lambda (p_M - \theta)}{\lambda K + (p_M - \theta) Q'_M} \]  

(A.16)

Because the shape of the participation set is sensitive to the size of the fixed cost \( F_m \), closed-form solutions are difficult to obtain. To understand the nature of this difficulty, let us assume for a moment that there is no fixed cost and focus on the region defined by the first-order condition with respect to price (18).

For the functional forms given in (A.5), the function representing the first-order condition of the unconstrained program (A.12) is a line of slope \( \lambda \) while that representing (A.13) is a line of slope \( \omega (1 + \lambda) / \lambda \). From the second-order condition (A.14), the crossing point between these two lines is an unconstrained welfare maximizer if \( \omega (1 + \lambda) / \lambda > \lambda \) and this is so independently of the value of the fixed cost \( F_m \).

Let us now examine the participation set for the relevant area where \( p_M - \theta \geq 0 \). With \( F_m = 0 \) the boundary of this set will be flat when \( (p_M - \theta) = 0 \) and will have a slope equal to \(-1\) when \( (p_M - \theta) > 0 \) and \( K = Q_M \) on this negatively-slopped portion of the boundary. Figures A2a and A2b below show these features. The shaded areas correspond to the participation set defined by (16) and the upward-sloping lines represent the first-order conditions (A.12) and (A.13).

**Figure A2a:** Interior solution with \( \omega(1 + \lambda) - \lambda^2 > 0 \)

**Figure A2b:** Boundary solution with \( \omega(1 + \lambda) - \lambda^2 < 0 \)

\[ 31 \text{The second-order conditions for this boundary solution are synthesized by} \]

\[
\begin{align*}
(1 + \lambda) (Q_M - (1 + \lambda) K)^2 C''(K) Q''_M &- \phi(Q_M - K) + \lambda K \left[ (2 \phi^2 + 3 \phi - 2 \sqrt{2})(Q_M - K) + \lambda (1 + 2 \lambda) K \right] Q'_M^2 \\
&+ \phi(Q_M - K) + \lambda K [Q''_M \leq 0]
\end{align*}
\]
In both of these figures the downward-sloping dashed line represents the \((K, p_M)\) pairs satisfying \([\theta Q' + (Q_M - K)] = 0\). Figures A3a and A3b below show the geometric characterization of the first-order condition with respect to price of the unconstrained program (A.12). Provided that \(\phi\) is nonnegative, the \((K, p_M)\) pairs satisfying (18) belong to the shaded areas in these figures. For alternative values of the cost-gap \(\theta - c\), we see from (19) that the solution of the constrained program lies on the bold segments shown in Figures A3a and A3b.

![Figure A3a: Locus of solutions with \(\omega(1 + \lambda) - \lambda^2 > 0\)](image1)

![Figure A3b: Locus of solutions with \(\omega(1 + \lambda) - \lambda^2 < 0\)](image2)

Now, when we proceed to generalize this argument to the case where \(F_m > 0\), the bold segments representing the solution to the constrained program in Figures A3a and A3b become curves, and more importantly, their shapes are sensitive to the size of the fixed cost. Figures A4a and A4b below show these bold curves for two different values of \(F_m\) with those on the upper parts corresponding to a lower fixed cost than those on the lower parts. Cross-examining Figures A2a-A2b and A3a-A3b, we see that when there is no fixed cost solutions with \(\phi > 0\) happen only in the negatively-sloped portion of the boundary of the participation set. In contrast, with a positive fixed cost a solution with \(\phi > 0\) may lie on either the positively- or negatively-sloped portion of the boundary of the participation set. This “indeterminacy” of the solution suggests that numerical and simulations methods may be appropriate for studying the behavior of the endogenous variables of this scheme \(p_M, K,\) and \(\phi\).
Proof of Proposition 2: Since $\theta - c > 0$, the functions representing (A.12) and (A.13) do not cross at $K = 0$, but we know that they cross at most once at a point where $K > 0$. However, a policy that prescribes $K = 0$ might still be optimal if $F_m$ is high enough to satisfy (22) with equality. These features characterize policy (i) given in the proposition.

If a crossing point of the functions representing (A.12) and (A.13) exists and belongs to the participation set (in which case $q_m > 0$), i.e., using (21) and (A.12), if $F_m < -\lambda K \frac{Q'_M(K)}{Q''_M(K)}$, this interior point, which from (A.13) is characterized by $(1+\lambda)\left[c + C'(K)\right] - \lambda p_M = \theta$, is picked up as the solution of the constrained welfare maximization program. Such crossing point is defined by $\lambda^2 K = (1+\lambda)Q'_M \left[\left[\theta - c\right] - C'(K)\right]$, rewritten as (26), which results from (A.12) and (A.13), rewritten as (24) and (25). Now, to guarantee that it exists, solving (26) for $\lambda^2$ and substituting into the second-order conditions (A.14) yields the technical condition $Q''_M(K) - \frac{C'(K)}{K} = \frac{\lambda K}{Q''_M(K)}Q''_M(K) < 0$. This characterizes policy (ii-a).

If the crossing point of the functions representing (A.12) and (A.13) does not exist or
lies outside the participation set, the optimization program picks the boundary solution satisfying (18), (19), and (A.16). These conditions are rewritten, respectively, as (27), (28) and (29). This corresponds to policy (ii-b).

Figures A5a-A5d illustrate these policies for specific functional forms in the $K-p_M$ space for the simplified case under which the cost gap ($\theta - c$) converges to cero.\[^{32}\]

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\[^{32}\]Figure A5a is based on the functional forms (A.5), $(\lambda, \omega, \gamma, \theta = c) \in \{(1/3, 1/2, 10, 2), (3/2, 1/2, 10, 2)\}$, respectively, and $F_m = 10.24$. Figures A5b and A5c employ the linear demand in (A.5), the capacity building cost function $C(K) = (\omega K + \gamma)K^2$, $(\lambda, \omega, \sigma, \gamma, \theta = c) \in \{(3/2, 1/2, 1/200, 10, 2), (3/2, 1/15, 1/200, 10, 2)\}$, and with $F_m = 3$ and $F_m = 10$, respectively. Finally, Figure A5d uses the specification and parameter values in Figure A5c with $F_m = 3$. 

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Controlling the regional monopoly with transport capacity only

Illustration of the program resolution approach: Turning to the study of the solution to the system (33)-(37) in the $K-p_M$ space, observe that since social welfare under this control scheme is the same as that in the previous section, so is the analysis of the unconstrained maximization program. When constraints are introduced in the maximization program, however, an additional one arises here, namely, the profit-maximization constraint (31). Such a constraint is represented in the $K-p_M$ space by a decreasing concave function with slope $-Q''_M/[Q_M - K]Q'_M - 2Q'_M^2$ and intercept point strictly in the interior of the participation set. Furthermore, this function crosses the boundary of the participation set at a point where the latter is infinitely sloped.\(^{33}\)

Equation (33), (34), and (37) define a tangency point between a welfare level curve and the function that represents the profit-maximization constraint. Hence, such a point satisfies

$$-\frac{Q'_M}{(Q_M - K)Q''_M - 2Q'_M^2} = \frac{(1 + \lambda)[(\theta - c) - C'(K)]Q'_M - \lambda(Q_M - K)}{[Q_M - (1 + \lambda)K]Q'_M} \quad (A.17)$$

If such a tangency point satisfies the firm’s participation constraint (36) with a strict inequality, it is an interior solution.\(^{34}\) Note from (A.17) that $K = 0$ cannot be a tangency point, and hence not an interior solution. If such a tangency point violates (36), the solution to (33)-(37) lies at the intersection of the function representing the profit-maximization constraint and the boundary of the participation set where, recall, the latter is infinitely sloped.\(^{35}\)

Let us illustrate the solution under this control scheme using the functional forms (A.5). In this case, the set defined by the firm’s profit maximization constraint (31) is a line of slope $-1/2$ that crosses the boundary of the participation set at the point where the latter is infinitely sloped, as shown in Figures A6a and A6b. The shaded regions correspond to the participation set defined by (30). The upward-sloping lines represent the price and capacity first-order conditions of the unconstrained program, respectively, (A.12) and (A.13). The downward-slopping dashed line is the set of $(K, p_M)$ pairs which satisfy the profit-maximization constraint of the local monopoly (31).

\(^{33}\)The reader can check that such a crossing point is characterized by the condition

$$F_m = 0 \quad (Q_M(p_M) - K)$$

Solving for $F_m$ and substituting into the expression of the slope of the boundary of the participation set (A.15) yields the slope of this set at the crossing point.

\(^{34}\)Second-order conditions are synthesized as:

$$-\Omega^2(1 + \lambda)C''(K) + 2\lambda\Omega - \left[\frac{(Q_M - K)(Q''_M - \eta Q''_M)}{Q'_M} + [Q'_M - 3\eta Q''_M]\right] < 0$$

Note that for a downward-sloping linear demand, the former condition holds for any value of $\eta$.

\(^{35}\)In this case, second-order conditions are always satisfied. It is worthwhile noting that (22) and (A.17) imply that transport capacity is always built under this scheme. This point will be further discussed in the next section.
Figure A6a sketches the case in which the solution lies in the interior of the participation set. This interior solution is
\[
K = \frac{(1 + 2\lambda)(\gamma - c) + (3 + 2\lambda)(\theta - c)}{1 + 4\lambda + 4\omega(1 + \lambda)}
\]
(A.18)
\[
p_M = \theta + \frac{[\lambda + 2\omega(1 + \lambda)](\gamma - c) - (\theta - c)[2 + 3\lambda + 2\omega(1 + \lambda)]}{1 + 4\lambda + 4\omega(1 + \lambda)},
\]
(A.19)
and emerges when the condition
\[
(\theta - c) < \frac{(\gamma - c)[\lambda + 2\omega(1 + \lambda)] - \sqrt{F_m}[1 + 4\lambda + 4\omega(1 + \lambda)]}{2 + 3\lambda + 2\omega(1 + \lambda)}
\]
(A.20)
holds. It represents policies of type (i) in the proposition. When condition (A.20) does not hold, the solution is on the boundary of the participation set. Figure A6b shows such a boundary solution given by
\[
K = \gamma - \theta - 2\sqrt{T_m}
\]
(A.21)
\[
p_M = \theta + \sqrt{T_m}
\]
(A.22)
This solution represents the policies of type (ii) in Proposition 3.

Proof of Proposition 3: Before sketching the proof, let us recall from our discussion that precedes the proposition in the text that \(K = 0\) is never a solution to the constrained welfare maximization program.

Since \((\theta - c) > 0\) and \(F_m > 0\), the capacity-price pair that maximizes the firm’s profit, defined by (37), belongs to the participation set if (36) holds with a strict inequality, i.e., if the fixed cost belongs to the interval \(F_m < \frac{(Q_m - K)^2}{Q_M}\). Since by definition an interior solution satisfies (A.17), which stems from (33), (34), and (37), pricing and capacity building obey (38)-(40). Finally, given that \(\phi = 0\), (34) can be rewritten as \((1 + \lambda)[c + C'(K)] - \lambda p_M = \theta + \eta\). This characterizes policy (i).
Proof of Proposition 4: As \( C'(K) \geq 0 \) and looking at (7) and (19), we have
\[
\text{sign}[K^B - K^A] = \text{sign}[(1 + \lambda)[C'(K^B) - C'(K^A)]]
\]
Given that \( \nu^A \geq 0 \), it follows from (A.23) that if \( (\lambda - \phi^B) > 0 \), \( \text{sign}[K^B - K^A] > 0 \). When \( (\lambda - \phi^B) < 0 \), we need to show that \( \text{sign}[K^B - K^A] < 0 \). We do so, by analyzing the optimal capacity level only in the case where \( \text{sign}[K^B - K^A] \) might be ambiguous, i.e., when both \( \phi^B \) and \( \nu^A \) are strictly positive.

If \( \phi^B > 0 \) and \( \nu^A > 0 \), from (44)-(47), \( \partial W^A/\partial p_M > 0 \), \( \partial W^A/\partial K^A > 0 \), and \( \partial W^B/\partial K^B > 0 \).

Under \( B \), the solution lies on the boundary of the participation set and is characterized by (A.16), rewritten as
\[
-\frac{\partial W^B}{\partial K} = \frac{\partial W^A}{\partial p_M} = \frac{\text{sign}(\phi^B - \theta)}{\partial W^B/\partial p_M}.
\]
Using (48) and (49), we obtain
\[
-\frac{\partial W^B}{\partial K} = -\frac{\partial W^A}{\partial p_M} = \frac{\text{sign}(\phi^B - \theta)}{\partial W^A/\partial p_M}.
\]
This says that at the point where the boundary of the participation set is tangent to a welfare level curve in \( B \), the former is also tangent to a welfare level curve in \( A \). Since \( \nu^A > 0 \), the solution under \( A \) has \( K^A = Q_M \) characterized by (A.4). Figure A7 illustrate this feature for the case where indeed \( F_m > 0 \).

\[\text{Figure A7:} \text{ Solutions with } \nu^A > 0, \phi^B > 0, \text{ and } F_m > 0\]

We know from section 4 that the participation set is included in the nonnegativity set for \( p_M \geq 0 \) (see Figure A7). Hence, any boundary solution under scheme \( B \) yields a level of capacity no greater than that under \( A \), i.e., \( K^B \leq K^A \).

\( ^{36}\)To simplify notation, from now on we define \( \partial W^i/\partial p_M \equiv \partial W^i/\partial p_M|_{p_M = p^*_M} \) and \( \partial W^i/\partial K^i \equiv \partial W^i/\partial K|_{K = K^i} \), for \( i \in \{A, B, C\} \).

\( ^{37}\)This figure employs the functional forms (A.5) with parameter values \( (\lambda, \omega, \gamma, \theta, c) = (1/3, 1/2, 10, 5, 2) \). The size of the fixed cost is \( F_m = 3 \).
Proof of Proposition 5: Direct comparison of the first-order conditions with respect to capacity under schemes B and C, (19) and (34), yields
\[
\text{sign}[K^C - K^B] = \text{sign}[(1 + \lambda)[C'(K^C) - C'(K^B)]]
\]
\[
= \text{sign}[(\lambda - \phi^C)(\frac{p_M^C - \theta}{\theta}) - (\lambda - \phi^B)(\frac{p_M^B}{\theta} - \theta) + \eta^C] \quad (A.24)
\]
We proceed to analyze the behavior of (A.24) for the possible realizations of $\phi^B$, $\phi^C$, and $\eta^C$, assuming first that the latter is positive and then negative.

If $\eta^C > 0$ and $\phi^C = 0$, we see from (51) and (52) that the constrained solution of $C$, which lies inside the participation set, satisfies $\partial W^B / \partial p_M^C < 0$, $\partial W^B / \partial K^C < 0$, and $\partial \Pi_m / \partial p_M^C = 0$.\(^{38}\) Two cases might arise according to whether or not $\phi^B$ is zero. First, when the solution to the constrained program under $B$ is interior, $\phi^B = 0$, $\partial W^B / \partial p_M^B = \partial W^B / \partial K^B = 0$. It is straightforward to see that $K_C^C > K_B^B$ and $p_M^C > p_M^B$. Second, when the solution to the constrained welfare maximization program under $B$ yields $\phi^B > 0$, we see from (47) that $\partial W^B / \partial K^B > 0$. Given that $\eta^C > 0$, this solution satisfies $\partial W^B / \partial p_M^B < 0$.\(^{39}\) Putting these properties together, we see from (46) that $\partial \Pi_m / \partial p_M^B > 0$, saying that at this boundary solution under $B$, firm’s marginal revenue is lower than its marginal cost. It then directly follows that $p_M^C > p_M^B$ and $K^C > K^B$.

If $\eta^C > 0$ and $\phi^C > 0$, we know from section 5 that the solution of the constrained program under $C$ is at the point where the boundary of the participation set is infinitely sloped, which has the largest $K$ of all the points in the participation set. From (51)-(53), the constrained solution of $C$ satisfies $\partial W^B / \partial p_M^C < 0$ and $\partial \Pi_m / \partial p_M^C = 0$. Again, two cases are to be considered. First, if the solution under scheme $B$ yields $\phi^B = 0$, since it lies in the interior of the participation set, it automatically implies a lower level of capacity than that under $C$, i.e., $K_C^C > K_B^B$. Second, if the solution of $B$ yields $\phi^B > 0$, we see from (47) that $\partial W^B / \partial K^B > 0$. From $\eta^C > 0$, it should be the case that $\partial W^B / \partial p_M^B < 0$. Using the latter inequality in (46) we obtain $\partial \Pi_m / \partial p_M^B > 0$, and hence $p_M^C > p_M^B$ and $K^C > K^B$.

To sum up, so far we have that when $\eta^C > 0$, $K^C > K^B$ and $p_M^C > p_M^B$. Let us now consider the cases where the Lagrange multiplier of the profit-maximization constraint $\eta^C$ is negative.

When $\eta^C < 0$ and $\phi^C = 0$, we see from (51)-(53) that the constrained solution of $C$ satisfies $\partial W^B / \partial p_M^C > 0$, $\partial W^B / \partial K^C > 0$, and $\partial \Pi_m / \partial p_M^C = 0$. Two cases arise depending on the sign of $\phi^B$. First, when $\phi^B = 0$, we directly see that $K_C^C < K_B^B$ and $p_M^C < p_M^B$. Second, when $\phi^B > 0$, the boundary solution satisfies $\partial W^B / \partial p_M^B > 0$, $\partial W^B / \partial K^B > 0$, and $\partial \Pi_m / \partial p_M^B < 0$. In this case, $(\partial W^B / \partial K^C - \partial W^B / \partial K^B) = -\eta^C - \phi^B(p_M^B - \theta) > 0$, and hence the capacity ranking is ambiguous. It is worth noting that while the capacity ranking is ambiguous, that of pricing is not. Indeed, since $\partial \Pi_m / \partial p_M^B < 0$, we obtain $p_M^C < p_M^B$.

When $\eta^C < 0$ and $\phi^C > 0$, we see from (51)-(53) that the constrained solution of $C$\(^{38}\)It is worthwhile noting that the existence of this type of solution depends on the fact that there exists a $K > 0$ satisfying the condition $Q_M \left[ C''(K) - \frac{C''(K)}{\alpha} \right] - \frac{\alpha}{Q_M} Q_M C'(K) < 0$. When the former condition is not satisfied, no solution with $\eta^C > 0$ and $\phi^C = 0$ exists.

\(^{39}\)If we assume that the boundary solution of $B$ lies on the region with $\partial W^B / \partial p_M^B > 0$, the tangency between a welfare level curve and the boundary of the participation set lies in their negatively sloped regions. By definition, when the boundary of the participation set is negatively sloped, it lies to the right of the function representing the profit-maximization constraint. Then, the solution to the constrained program in $C$ should be characterized by $\partial W^B / \partial p_M^B > 0$. But, this contradicts $\eta^C > 0$.
satisfies $\partial W^B/\partial p^C_M > 0$, $\partial W^B/\partial K^C > 0$, and $\partial \Pi_m/\partial p^C_M = 0$. The only case to be analyzed then is when $\phi^B > 0$. Under this case, since $\eta^C < 0$, we have $\partial W^B/\partial p^B_M > 0$, $\partial W^B/\partial K^B > 0$, and $\partial \Pi_m/\partial p^B_M < 0$. In this case, $(\partial W^B/\partial K^C - \partial W^B/\partial K^B) = -\eta^C - \phi^B(p^B_M - \theta) + \phi^C(p^C_M - \theta) \geq 0$, and again the capacity comparison is ambiguous. Note that from $\partial \Pi_m/\partial p^B_M < 0$, we obtain $p^C_M < p^B_M$. In this case, $(\partial W^B/\partial K^C - \partial W^B/\partial K^B) = -\eta^C - \phi^B(p^B_M - \theta) + \phi^C(p^C_M - \theta) \geq 0$, and again the capacity comparison is ambiguous. Note that from $\partial \Pi_m/\partial p^B_M < 0$, we obtain $p^C_M < p^B_M$.

Summarizing, we see that when $\eta^C < 0$, $p^C_M < p^B_M$, but the capacity ranking remains undetermined. ■

Proof of Proposition 6: A cross-examination of the first-order conditions with respect to capacity under schemes $A$ and $C$, (7) and (34), shows that

$$
sign[K^C - K^A] = sign[(1 + \lambda)[C'(K^C) - C'(K^A)]]
= sign[(\lambda - \phi^C)(p^C_M - \theta) + \eta^C + \nu^A]
$$

(A.25)

We proceed to analyze the behavior of (A.25) for the possible realizations of $\nu^A$, $\phi^C$, and $\eta^C$, assuming first that the latter is positive and then negative. If $\eta^C > 0$ and $\phi^C = 0$, from (55) we get that the solution under $C$ satisfies $\partial W^A/\partial K^C = -\lambda(p^C_M - \theta) - \eta^C < 0$. The only case that is relevant to examine is when $\nu^A = 0$.41 In this case, $\partial W^A/\partial K^A = 0$ and then it is easy to see that $K^C > K^A$.

If $\eta^C > 0$ and $\phi^C > 0$, two cases might arise according to whether or not $\nu^A = 0$. When $\nu^A = 0$, we see from (55) that the solution under $C$ satisfies $\partial W^A/\partial K^C = -(\lambda - \phi^C)(p^C_M - \theta) - \eta^C \geq 0$, and hence the capacity comparison is undetermined. When $\nu^A > 0$, the solution under $A$ satisfies $K^A = Q_M$ and we directly conclude that $K^C \leq K^A$.

Summarizing, we have obtained that $sign[K^C - K^A] = sign[\eta^C]$ when $\eta^C > 0$ but only in the case where $\phi^C = 0$. Let us now consider the case where the Lagrange multiplier of the profit-maximization constraint $\eta^C$ is negative. If $\eta^C < 0$ and $\phi^C \geq 0$, the capacity ranking is undetermined since the solution under $C$ satisfies $\partial W^A/\partial K^C = -(\lambda - \phi^C)(p^C_M - \theta) - \eta^C \geq 0$. ■

40 Indeed, the largest $K$ attained by the participation set is strictly lower than that obtained from the condition $\partial W^B/\partial p^B_M = \partial W^B/\partial K^B = 0$ which defines an interior solution under $B$.

41 Indeed, if $\nu^A > 0$, we see from (45) and (55) that $\eta^C = -\nu^A - \lambda(p^A_M - \theta)$. Since $\phi^C = 0$, it should be the case that under $A$, $p^A_M > \theta$ and hence $\eta^C < 0$, a contradiction.
References


