POLITICAL INTERVENTION IN ECONOMIC ACTIVITY

Enrique Gilles
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Abstract

This paper proposes a political economy explanation of bailouts to declining industries. A model of probabilistic voting is developed, in which two candidates compete for the vote of two groups of the society through tactical redistribution. We allow politicians to have core support groups they understand better, this implies politicians are more or less effective to deliver favors to some groups. This setting is suited to reproduce pork barrels or machine politics and patronage. We use this model to illustrate the case of an economy with both an efficient industry and a declining one, in which workers elect their government. We present the conditions under which the political process ends up with the lagged-behind industry being allowed to survive.

JEL Codes: D72, D78.

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1 Introduction

One of the first lessons of Economics refers to an infinity of agents taking economic decisions based on all the relevant information: this is our first approach to the perfect competition hypothesis. In the particular case of firms, the lesson continues, we learn they take prices as given, there are many of them, that their only goal is to maximize their profits, and that they are free to enter and leave markets. We soon realize these conditions are too strict, and that it is hard to find a perfectly competitive market. We then start learning about monopolies, oligopolies, and about barriers to entry: everything becomes clearer. Perfect competition also implies that only the most efficient firms can survive the harshness of markets, while the others are condemned to disappear.

This paper is about why those other firms are sometimes not allowed to fail. More precisely, we analyze government intervention that prevents firms to die: this constitutes another departure from the perfect competition hypothesis. The political intervention into economic activity is closely related to the soft budget constraint syndrome, i.e. an economic disease –deeply rooted in former socialist and transitional countries– that is characterized by persistent rescues of failing institutions by a support institution. The syndrome is fully at work when this intervention creates in turn expectations of future bailouts\(^1\).

The softening of budget constraints is not exclusive of transition economies. Take the case of the US automobile industry: there is a consensus around the fact that their financial troubles are closely related to the persistence of poor business practices (lack of innovation mainly) together with a unionized workforce. These problems have been arising since the late seventies, and their partial resolution has often implied government’s transfers. This support has crystallized under different forms: subsidized credit, bridge loans, and backing of warranties. Thirty years later, we witness the end of an era in the American automobile industry, and there is enough evidence to assert that repeated bailouts have had a negative impact on the incentives to innovate of these firms. In turn, successive governments, either captured by interest groups or invoking the need to protect employment, have all shown some degree of willingness to intervene.

\(^1\)For an extensive review on the literature of the soft budget constraint, see Kornai et al (2003).
Hence, if we are to understand the reasons of the persistence of government support and the survival of an inefficient industry, we cannot avoid discussing political factors such as rent seeking, lobbying activities and the political power of concerned groups. Politicians take economic decisions, and often these decisions do not pursue economic efficiency but they reflect the political power of interest groups.

We present a model of probabilistic voting to address this issue of political intervention in economic activity. We identify the conditions under which two interest groups may benefit or suffer from redistribution schemes as a result of the political game between two office-motivated candidates. These candidates are characterized by different abilities to deliver political favors to interest groups.

Dixit and Londregan (1996) analyze the main characteristics that special interest groups may have in order to be benefitted by redistribution in electoral contests. They identify those traits in two settings: if parties have no differences in their ability to deliver favors, the benefits of redistribution will be mainly targeted to swing voters. On the other hand, with different abilities to transfers, parties will favor their core support groups. These abilities are synthetized in the leaky bucket assumption: there are deadweight losses in the redistribution activity, that depend on parties’ proximity to voters. When parties have core support groups they understand better, they can more easily target subsidies to them in exchange of votes. It is then a rationale for “pork barrel” politics, “machine politics”, or “patronage”, that is, spending that is intended to benefit constituents of a politician in return for their political support, either in the form of campaign contributions or votes.

In a related work, Dixit and Londregan (1995) address the issue of redistributive politics and economic efficiency. Using a probabilistic model they show that workers in a lagged-behind industry benefitting from subsidies may have no incentives to relocate to others, more efficient industries. Then inefficient industries may be totally locked in: their workers know that if they move nothing guarantees they will continue to be recipients of political redistribution. So even if the decision to move would imply a higher labor income, this advantage may vanish when political considerations are taken.

Robinson and Torvik (2005) propose a political economy model to explain the existence and persistence of soft budget constraints. Their starting argument is that an
incumbent politician may be willing to launch projects known to be inefficient, provided only him is able to refinance them tomorrow, i.e. after elections. This creates incentives for some voters to vote for him, since it is the only way their projects will be bailed out. The soft budget constraint syndrome is not viewed as in the standard way, i.e. a problem, but as an opportunity for politicians to get electoral support. The approach followed in the paper is based on a probabilistic model and using the leaky bucket assumption, but in a simplified way than the one we develop here.

In this paper we take the leaky bucket assumption of Dixit and Londregan (1996) model and, by focusing on two special interest groups, a given utility function and a distribution of ideology among groups, we are able to obtain a closed form solution for the redistributive scheme of each candidate. Furthermore, we show that under different candidates’ abilities to redistribute, parties platforms differ. Usually, the equilibrium in probabilistic voting models entails both parties proposing the same policies. However, we should recognize that this is hardly found in real life politics (e.g. conservatives are more pro-market whereas liberals are more interventionists, and this surely implies different policies). Our model is capable to reproduce this divergence, but in order to know which is the implemented policy, it is relevant to know who is the winner of elections in a given political equilibrium. We derive conditions to identify the winner.

We then setup an example in which a lagged-behind industry is allowed to survive given that its workers are key in electoral terms to one of the politicians. In this sense, we offer a model that sheds some light, from a political economy perspective, in explaining the occurrence and persistence of bailouts.

The paper is presented as follows: in section 2 we present the political equilibrium of the model. Section 3 presents the economic model. Section 4 concludes.

2 Political equilibrium

Consider an economy in which the society is represented by two groups, indexed by $i = \{1, 2\}$. Group 1 has $N_1$ members whereas the number of individuals in group 2 is $N_2$. Since we are not concerned with the decision of whether to vote, we assume each group member actually votes, so we are calling them voters (and also, for the purposes
of the next section, workers). The total vote is thus equal to the total population $N = N_1 + N_2$.

There are 2 political parties (also called candidates) denoted by $k = \{A,B\}$. These candidates compete to attract voters in order to maximise their total vote, by proposing transfers to each group. Their motivation is only to be in power (from which they may extract exogenous rents) and they do not have partisan preferences. However, they do have some observable characteristics to voters, namely an issue position, ideology or even popularity. In any case, these attributes are considered as fixed so that they cannot be accommodated during a given electoral process. Parties platforms consist in a redistributive scheme to group members, which is implemented with certainty if the party wins the elections. This last assumption on enforceability will allow us to name parties indistinctly as “candidate $k$” or, in some contexts, simply “the government”. Platforms are denoted by $T_{ik}$, that is the amount of redistribution from party $k$ to each of group $i$ members.

Leaky bucket assumption: the technology to collect taxes and pay subsidies implies that there are leakages between what the government collects ($T_{ik}$) and what agents receive ($t_{ik}$):

$$t_{ik} = \begin{cases} (1 + \gamma_{ik})T_{ik} & \text{if } T_{ik} < 0 \\ (1 - \theta_{ik})T_{ik} & \text{if } T_{ik} > 0 \end{cases}$$

(1)

Where the (positive) parameters $\gamma_{ik}$ and $\theta_{ik}$ indicate the ability party $k$ has to tax or transfer money to group $i$, respectively. Note these abilities may be asymmetric, i.e. a candidate may be skilled in transferring to some particular group and at the same time have less ability to tax the other group, or viceversa. Parameter values close to zero mean a close relationship between politicians and group members, while high values indicate the opposite. Parties have the following budget constraint$^2$:

$$\sum_i N_i T_{ik} = 0$$

(2)

The level of income $\omega_{ik}$ of an individual in group $i$ when candidate $k$ is on office, is

$^2$We could have a positive amount available for redistribution, at a higher cost in terms of algebra.
composed by wages and transfers:

\[ \omega_{ik} = w_i + t_{ik} \]  

Wages are group-specific and they are set as the marginal productivity of labor. Individuals derive utility from income, their indirect utility function is given by:

\[ U_i(\omega_{ik}) = \sigma \ln \omega_{ik}, \quad 0 < \sigma < 1 \]

The sequence of events in this game is the following: first, parties choose their platforms \( \{T_{ik}\} \), elections take place, and finally the winner implements its policy.

In deciding their vote, workers care not only about economics but also about ideology. This means they are sensitive to transfers but they also have an issue position. We rule out the possibility to interpret group membership as deriving from ideological aspects, instead, we allow for political diversity within each group. We assume that ideology is continuously distributed among voters of each group, such that a voter located at \( X \) has an ideological preference \( X \) for candidate \( B \).

The voting decision is therefore the following: a worker of group \( i \) votes for candidate \( A \) if the indirect utility she gets should \( A \) win more than compensates the utility she would get if \( B \) wins, taking into account her ideological preferences over \( B \), i.e.

\[ \text{vote for } A \iff U_i(\omega_{iA}) > U_i(\omega_{iB}) + X. \]

Given the distribution of ideology among voters, we can define the marginal voter in group \( i \), that is, the one who is just indifferent between voting for candidate \( A \) and candidate \( B \): this indifferent voter is characterized by having an ideology bias towards candidate \( B \) that we denote \( X_i \), defined then as

\[ X_i \equiv U_i(\omega_{iA}) - U_i(\omega_{iB}). \]

All individuals in group \( i \) characterized by an ideology level \( X < X_i \) vote for candidate \( A \). Taking into account our specific indirect utility function, this cutpoint is:

\[ X_i \equiv \sigma \ln \omega_{iA} - \sigma \ln \omega_{iB} = \sigma \ln \left( \frac{\omega_{iA}}{\omega_{iB}} \right). \]
We assume we know the distribution of the exponential of $X$,

$$e^X \equiv x \sim U \left[ 1 - \frac{1}{2\delta_i}, 1 + \frac{1}{2\delta_i} \right]$$

where $\delta_i$ is the probability density function of $x$ in group $i$. Then the ideology cutpoint is

$$x_i = \left( \frac{\omega_i A}{\omega_i B} \right)^{\sigma}.$$  

Letting $\Phi_i(x)$ be the cumulative distribution function of $x$ in group $i$, the proportion of members of group $i$ voting for party $A$ is given by:

$$\text{Prob}_i(x \leq x_i) = \Phi_i(x_i) = \delta_i(x_i - 1) + \frac{1}{2}$$

whereas the proportion of group $i$ individuals voting for $B$ is

$$\text{Prob}_i(x \geq x_i) = 1 - \Phi_i(x_i) = \frac{1}{2} - \delta_i(x_i - 1)$$

we have

$$\frac{\partial\Phi_i(x_i)}{\partial x_i} = \phi_i(x_i) \equiv \delta_i$$

$$\Phi_i(1) = \frac{1}{2} \quad \text{and} \quad E(x) = 1.$$  

From the above facts, when parties choose their platforms the thresholds $x_i$ are determined, and we can express the total vote for each party as

$$V_A = \sum_{i=1}^{i=2} N_i \Phi_i(x_i),$$

and

$$V_B = \sum_{i=1}^{i=2} N_i [1 - \Phi_i(x_i)] = N - V_A.$$  

2.1 Which group will be benefitted by redistribution?

Both parties choose their platforms $\{T_{ik}\}$ so as to maximize their total vote $V_k$. As the budget constraint -equation (2)- indicates, if parties make positive transfers to a group, the other group is necessarily being taxed. The problem at this point in the analysis
is to find under which conditions a given group will be taxed while the other gets a subsidy. Which are the main features that an interest group must have in order to profit from redistribution? Let us define the following variables that help deal with the leaky bucket parameters:

when party $k$ taxes group 1 and subsidizes group 2, we use: 
\[ \psi_k = \frac{1 - \theta_{2k}}{1 + \gamma_{1k}} \]

when party $k$ taxes group 2 and subsidizes group 1, we use: 
\[ \epsilon_k = \frac{1 - \theta_{1k}}{1 + \gamma_{2k}} \]

See that the maximum value of $\psi_k$ and of $\epsilon_k$ is one, corresponding to the case $\theta_{ik} = \gamma_{ik} = 0$, that is, when there are no leakages between the government and the groups. Thus, the closer $\psi_k$ and $\epsilon_k$ to unity, the more ability party $k$ has in the redistributive activity.

**Proposition 1** Groups benefitted from redistribution

- **Party A taxes group 1 to subsidize group 2 if** 
  \[ \frac{\delta_2 w_1}{\delta_1 w_2} > \frac{\psi_B^\sigma}{\psi_A^\sigma} \]

- **Party A taxes group 2 to subsidize group 1 if** 
  \[ \frac{\delta_2 w_1}{\delta_1 w_2} < \frac{\epsilon_A^{1+\sigma}}{\epsilon_B^{1+\sigma}} \]

- **Party B taxes group 1 to subsidize group 2 if** 
  \[ \frac{\delta_2 w_1}{\delta_1 w_2} > \frac{\psi_B^{-1}}{\psi_A^{-1}} \]

- **Party B taxes group 2 to subsidize group 1 if** 
  \[ \frac{\delta_2 w_1}{\delta_1 w_2} < \epsilon_A^\sigma \epsilon_B^{1-\sigma} \]

*Proof. See the Appendix.*

We show these results in Figure 1. Group $i$ will be benefitted the lower its wage $w_i$ and the higher the density $\delta_i$ compared to the respective values of the other group. In other words, parties will tax the group with higher pre-tax income, and they will offer subsidies to groups with a high density. Note the uniform distribution assumption implies a constant density and this simplifies the analysis. The condition actually says that benefitted groups have a high density at the indifferent voter, i.e. a high $\phi_i(x_i)$. Therefore, parties will favor groups with a higher proportion of those marginal voters, characterized by a higher willingness to trade ideology for transfers (i.e. more center in the political spectrum). Another important result is that the number of members of a group has no influence on the determination of the favored group. This means
that even a small group may be benefitted. Note that for some levels of wages and densities, parties do not propose any transfer. Finally, we also show in the Appendix that strategies are characterized by both parties taxing or subsidizing the same groups, i.e. we cannot find a situation in which, say, party A taxes group 1 while party B taxes group 2.

\[
\begin{array}{c}
A \text{ taxes group 2} \\
A \text{ taxes group 1} \\
B \text{ taxes group 2} \\
B \text{ taxes group 1}
\end{array}
\]
\[
\begin{array}{c}
\epsilon_A \left( \frac{\psi_A}{\psi_B} \right)^\sigma \\
\frac{1}{\psi_A} \left( \frac{\psi_B}{\psi_A} \right)^\sigma \\
\epsilon_B \left( \frac{\psi_A}{\psi_B} \right)^\sigma \\
\frac{1}{\psi_B} \left( \frac{\psi_B}{\psi_A} \right)^\sigma
\end{array}
\]

\[
\frac{\delta_2 w_1}{\delta_1 w_2} > \left( \frac{\psi_B}{\psi_A} \right)^\sigma \max \left\{ \frac{1}{\psi_A}, \frac{1}{\psi_B} \right\}
\]

(9)

A sufficient condition under which both parties tax group 1 and subsidize group 2 is

i.e. if \( \psi_A < \psi_B \) and if party A taxes group 1 to subsidize group 2, then party B do the same. If \( \psi_A > \psi_B \) the opposite holds.

2.2 The political game

Let us consider hereafter the case in which both parties decide to tax group 1, using these proceeds to subsidize group 2. As seen above, this corresponds to a situation in which group 1 wage is high with respect to the wage in group 2 and when \( \delta_2 \) is high compared to \( \delta_1 \). Therefore we use the parameters embedded in \( \psi_k \) instead of those of \( \epsilon_k \).

Each party wants to maximize its total vote, \( V_k \), by choosing its platform \( \{ T_{ik} \} \), given the platform the other candidate has selected. Note that, since parties have a budget constraint, the problem is unidimensional: once \( T_{2k} \) has been chosen, \( T_{1k} \) is determined. Choosing platforms determines the income levels \( \omega_{ik} \) that group members will obtain under each candidate, see equation (3). Then the thresholds \( x_i \) —which are the main
inputs of the objective (vote) functions— are obtained. Finally, we get the total vote for each party.

The problem of Party $A$ is then the following:

$$
\begin{align*}
\max_{T_{1A}} & \quad V_A = \sum_{i=1}^{i=2} N_i \Phi_i(x_i) \\
\text{s.t.} & \quad \sum_{i=1}^{i=2} N_i T_{1A} = 0 \quad BC_A \\
& \quad \omega_{1A} = w_1 + t_{1A} > 0 \quad LL_{1A} \\
& \quad \omega_{2A} = w_2 + t_{2A} > 0 \quad LL_{2A}
\end{align*}
$$

Party $A$ seeks to maximize its total vote subject to the budget constraint ($BC_A$) and two limited liability constraints ($LL_{iA}$) to prevent agents get a non-positive income. Substituting the expressions for $x_i$, given by equation (4) and for $\Phi_i(x_i)$ given by equation (5), using the constraint $BC_A$ to eliminate $T_{1A}$, the problem may be specified as

$$
\begin{align*}
\max_{T_{2A}} & \quad V_A = \frac{N_1}{\sigma} + N_1 \delta_1 \left[ \left( \frac{\omega_{1A}}{\omega_{1B}} \right)^\sigma - 1 \right] + N_2 \delta_2 \left[ \left( \frac{\omega_{2A}}{\omega_{2B}} \right)^\sigma - 1 \right] \\
\text{s.t.} & \quad \omega_{1A} > 0 \quad LL_{1A} \\
& \quad \omega_{2A} > 0 \quad LL_{2A}
\end{align*}
$$

The first order conditions of this problem yield the best response function of party $A$ with respect to party $B$. We can express it as

$$
\chi_A = \left[ \frac{\delta_1}{\delta_2 \psi_A} \right]^{\frac{1}{1-\sigma}} \left[ \frac{1}{\chi_B} \right]^{\frac{\sigma}{1-\sigma}}
$$

where, for convenience, we use the variable $\chi_k$ defined as the following ratio:

$$
\chi_k = \frac{\omega_{1k}}{\omega_{2k}}
$$

and recall the levels of after-transfer income write

$$
\begin{align*}
\omega_{1k} &= w_1 - (1 + \gamma_{1k}) \frac{N_2}{N_1} T_{2k} \\
\omega_{2k} &= w_2 + (1 - \theta_{2k}) T_{2k}
\end{align*}
$$

The important thing to keep from this is each $\chi_k$ defines a unique value of $T_{2k},$

$$
T_{2k} = \frac{N_1 (w_1 - w_2 \chi_k)}{(1 - \theta_{2k}) N_1 \chi_k + (1 + \gamma_{1k}) N_2},
$$
with low values of $\chi_k$ corresponding to high values of the transfer to group 2 members, $T_{2k}$. Moreover, through the budget constraint, $\chi_k$ also defines a unique $T_{1k}$.

Similarly, party $B$’s problem is to maximize its total vote, which, following similar steps as done for $A$ results in the following problem depending on $T_{2B}$:

$$\max_{\{T_{2B}\}} \quad V_B = \frac{N}{2} - N_1\delta_1 \left[ \left( \frac{\omega_1A}{\omega_1B} \right)^\sigma - 1 \right] - N_2\delta_2 \left[ \left( \frac{\omega_2A}{\omega_2B} \right)^\sigma - 1 \right]$$

s.t. \quad $\omega_1B = w_1 + t_{1B} > 0$

$\omega_2B = w_2 + t_{2B} > 0$

The resulting best response function is given by:

$$\chi_B = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\psi_B} \right]^{\frac{1}{1-\sigma}} \chi_A^{\frac{\sigma}{1-\sigma}} \quad (13)$$

**Behavior of best response functions.** Figure 2 shows the best response functions of each party, together with the effects of a decrease in parties’ abilities to transfer funds. These lower levels of abilities are modelled through an increase of the parameters $\theta_{2A}$ and $\theta_{2B}$ (or also by an increase in the $\gamma_{1k}$ parameters). The bold lines represent the new position of the best response functions. For party $A$, the new best response functions lies to the north-east of the original one, while for party $B$ it lies to the north-west. Since the leakage occurred between the government and its beneficiaries diminishes the political power of the former to attract voters, candidates are better off with low parameter values. From this graphical analysis it follows that party $A$ will prefer low leaky-bucket parameter values such that its best response function lies the closer as possible to the origin. Similarly, party $B$ will prefer a best response function as closer to the horizontal axis as possible, corresponding to low values of the leaky-bucket parameters.

### 2.3 Equilibrium

The Nash equilibrium of this game is found at the intersection of both best response functions, equations (10) and (13):

$$\chi_A^*(\chi_B^*) = \chi_B^*(\chi_A^*)$$

11
Figure 2: Effects of a decrease in parties’ abilities to transfer

where starred variables denote equilibrium values. This equilibrium is then

$$\chi^*_A = \frac{\delta_1}{\delta_2} \left[ \frac{1}{\psi_A} \right]^{1+\sigma} \psi_B$$, (14)

$$\chi^*_B = \frac{\delta_1}{\delta_2} \left[ \frac{1}{\psi_A} \right]^{\sigma} \left[ \frac{1}{\psi_B} \right]^{1-\sigma}$$, (15)

Once the equilibrium values for these variables obtained, recalling equation (11), we immediately get the corresponding equilibrium transfers $T^*_{2k}$,

$$T^*_{2A} = \frac{N_1}{1-\theta_{2A}} \left[ \delta_2 \psi_A^{1+\sigma} w_1 - \delta_1 \psi_B^\sigma w_2 \right] \frac{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma}$$ (16)

$$T^*_{2B} = \frac{N_1}{1-\theta_{2B}} \left[ \delta_2 \psi_B^{1+\sigma} w_1 - \delta_1 \psi_B^\sigma w_2 \right] \frac{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma}$$ (17)

Figure 3 shows both candidates’ best response functions and the resulting Nash equilibrium, for the case in which party $A$ is more efficient in redistribution than $B$, i.e. for $\psi_A > \psi_B$.

We may next obtain the equilibrium levels of after-transfers income, $\omega_{ik}$:
Figure 3: Nash Equilibrium – party A more efficient in redistribution

\[ \begin{align*}
\omega_{1A}^* &= \frac{\delta_1 \psi_A^{-1} \psi_B^\sigma}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma} (N_1 \psi_A w_1 + N_2 w_2) \\
\omega_{2A}^* &= \frac{\delta_2 \psi_A^\sigma}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma} (N_1 \psi_A w_1 + N_2 w_2) \\
\omega_{1B}^* &= \frac{\delta_1 \psi_B^{-1}}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma} (N_1 \psi_B w_1 + N_2 w_2) \\
\omega_{2B}^* &= \frac{\delta_2 \psi_A^\sigma}{N_1 \delta_1 \psi_B^\sigma + N_2 \delta_2 \psi_A^\sigma} (N_1 \psi_B w_1 + N_2 w_2). 
\end{align*} \]

It is interesting to remark some facts about the equilibrium levels of income. To do that, let us first consider the case in which there are no leakages in redistribution such that \( \psi_k = 1 \). The after-transfers income of an individual of group \( i \) may be written as follows:

\[ \omega_{ik}^* = \frac{\delta_i}{\sum_i N_i \delta_i} Y \]
where $Y$ stands for the total income$^3$ of the economy, that is: $Y = N_1w_1 + N_2w_2$. The after-transfers income an individual of group $i$ gets from party $k$ is then some fraction of the total income. This result can be interpreted as if the government collected all the income to then redistribute it to group members according to some weights, and these weights reflect the political power of a given group. Group $i$’s members will be benefitted the higher their density $\delta_i$, that is, the more centered the distribution of ideology among voters. We have already mentioned a similar intuition when showing the features of benefitted groups by redistribution. Also, group size plays no direct role on determining the political power of a given group, but it has an indirect effect through the total income available for redistribution in the economy. Moreover, the pre-transfers level of income (i.e. wages) of an individual is almost irrelevant to determine her after-transfers income.

Things get less straightforward when we introduce the leaky bucket parameters. However, we can highlight a similar intuition: the total income available to candidate $k$ for redistribution is now

$$Y_k = N_1\psi_kw_1 + N_2w_2$$

(22)

this can be interpreted as the maximum amount of income candidate $k$ has in order to redistribute from group 1 to group 2. Indeed, suppose group 1 is fully taxed, then the proceeds candidate $k$ gets from taxing this group are

$$\frac{N_1w_1}{1 + \gamma_{1k}}$$

that is lower than $N_1w_1$ due to the leakages in the way from group 1 to party $k$. This amount would then serve to make transfers to the other group. Group 2 members would receive, after the leaky-bucket losses

$$\frac{N_1w_1}{1 + \gamma_{1k}}(1 - \theta_{2k}) \equiv N_1\psi_kw_1$$

which, in addition to their own income $N_2w_2$ makes the total after-transfers income available to candidate $k$ for redistribution ($Y_k$), for the case in which group 1 is taxed to subsidize group 2. In turn, the weights that determine the allocation of that income

$^3$In the absence of leakages and of any extra budget available for redistribution, the distinction between before and after transfers is not relevant.
to the different groups take into account now the effects of the leakages accruing in redistribution and, despite being not as much clear as for the no-leakages case, preserve the same qualitative insights: benefitted groups are those with a higher density $\delta_i$ and a better proximity to candidate $k$, i.e. a higher $\psi_k$.

Concerning the levels of the final income of each group member compared to their pre-transfers levels, we have the following result.

**Proposition 2** Groups with higher pre-transfer income may get a lower after-transfer income, i.e. after the political process.

*Proof.* Let us consider the case in which group 2 members get a higher income after the political process, i.e. $\omega^*_2 > \omega^*_1$ despite having a lower wage than group 1 members. The condition to have this is obtained from the equilibrium values of final income given by equations (18) and (19):

$$\omega^*_2 > \omega^*_1 \iff 1 > \frac{\delta_1 \psi_B^*}{\delta_2 \psi_A^{1+\sigma}}$$

We have now to check that this condition satisfies the requirements we have imposed to have party A taxing group 1 to subsidize group 2. Recalling Proposition 1, we need

$$\frac{w_1}{w_2} > \frac{\delta_1 \psi_B^*}{\delta_2 \psi_A^{1+\sigma}}$$

Therefore it may be the case that

$$\frac{w_1}{w_2} > 1 > \frac{\delta_1 \psi_B^*}{\delta_2 \psi_A^{1+\sigma}}$$

The procedure if we consider party B transfers is analogous. \(\square\)

The political characteristics of groups determine which group will be taxed and which will be subsidized. This result goes beyond that and better identifies winners and losers from the electoral process: in the example, the high-wage agents (i.e. group 1 members) not only subsidize the low-wage ones, but they also end up with a lower income, after transfers are made. This situation may arrive if the poor are a more ideology-concentrated group than the rich (i.e. if they belong to the center of the ideological spectrum).
With equations (18) to (21), we finally determine the equilibrium values of the cut-points $x_1$ and $x_2$,

\[
x_1^* = \left( \frac{\omega_{1A}^*}{\omega_{1B}^*} \right) = \left( \frac{\psi_B Y_A}{\psi_A Y_B} \right)^\sigma \\
x_2^* = \left( \frac{\omega_{2A}^*}{\omega_{2B}^*} \right) = \left( \frac{Y_A}{Y_B} \right)^\sigma
\]

It immediately follows that the following relationship holds

\[
x_1^* = \left( \frac{\psi_B}{\psi_A} \right)^\sigma x_2^*
\] (23)
Analysis of the equilibrium:

**Proposition 3** Under identical candidates’ abilities to transfer, i.e. \( \theta_{2A} = \theta_{2B} \) and \( \gamma_{1A} = \gamma_{1B} \) we find the standard political equilibrium result in which parties’ platforms are identical.

**Proof.** When \( \theta_{2A} = \theta_{2B} \) and \( \gamma_{1A} = \gamma_{1B} \) we have \( \psi_A = \psi_B = \psi \). We first insert \( \psi \) into the equilibrium values of \( \chi_A \) and \( \chi_B \) given respectively by equations (14) and (15). At this point we have \( \chi^*_A = \chi^*_B = \chi^* \). Then, the result follows from taking into account that each \( \chi_k \) defines a unique \( T_{2k} \) (and hence a unique \( T_{1k} \)) as equation (12) shows. \( \square \)

This case reproduces the convergence result of most political economy models, while in this setting it is just a particular case.\(^4\) See Figure 4. Instead, if leaky bucket parameters are such that \( \psi_A > \psi_B \) we lose platform convergence and the equilibrium is found above the 45\(^{\circ}\) line, as the example of Figure 3 showed. In such a setting party \( A \) is able to extract more funds from group 1 and to subsidize more efficiently group 2 than party \( B \). The opposite situations emerges whenever \( \psi_B > \psi_A \), in which the equilibrium lies below the 45\(^{\circ}\) line.

### 2.4 Election’s result

As seen above, given the different abilities parties have to redistribute to interest groups, their platforms will in general differ. In order to determine which is the implemented platform, we need to know who is the winner of the election. Recalling that every agent of the economy votes, we propose the following result:

**Proposition 4** In the case both parties tax group 1 to subsidize group 2, the party with the highest ability to transfer to groups (i.e. with the highest \( \psi_k \)) wins the elections. If both parties have the same ability to transfer (i.e. \( \psi_A = \psi_B \)) there is a tie in elections.

\(^4\)There is also the situation in which \( \theta_{2A} \neq \theta_{2B} \) and \( \gamma_{1A} \neq \gamma_{1B} \) but the parameter levels still imply \( \psi_A = \psi_B \) : Party platforms here differ but we have \( \chi^*_A = \chi^*_B = \chi^* \) as before.
\[ \omega_1 \beta \omega_2 \beta \]

\[ \chi_B(\chi_A) \]

\[ \chi_A(\chi_B) \]

\[ \chi^* \]

\[ \chi^* \]

Figure 4: Parties are equally efficient in redistribution

Proof. See the appendix.

3 The Economic model

In this section we propose an application of the political model introduced in the previous section. We now turn to the description of the economy. Groups 1 and 2 become here the workers of two sectors or industries of this economy: industry 1 and industry 2. We can think on sector 1 as being the new, highly productive industry. Sector 2 instead is the old, low productive and declining one. Workers have in principle the possibility to migrate from one sector to the other. However, we assume that in the short run there are switching costs sufficiently high so as to impede workers to do that. Hence, there is no labor mobility across sectors.

**Sector 1 description** The technology of sector 1 is represented by the following production function

\[ Y_1 = F_1(N_1) = A_1 N_1, \quad A_1 > 0, \]

where \( A_1 \) is the marginal productivity of labor in sector 1. Wages in this industry,
noted by $w_1$, are set as the marginal productivity of labor. The after-transfers income $\omega_{1k}$ of an individual working in this industry is given by this wage and the transfer she pays to the government in case candidate $k$ is in office

$$\omega_{1A} = w_1 + t_{1A} = w_1 - (1 + \gamma_{1A}) \frac{N_2}{N_1} T_{2A}$$

$$\omega_{1B} = w_1 + t_{1B} = w_1 - (1 + \gamma_{1B}) \frac{N_2}{N_1} T_{2B}$$

In sum, a little changes with respect to the canonical model presented in the preceding section.

**Sector 2 description** The production function is

$$Y_2 = F_2(N_2) = A_2 N_2, \quad A_2 > 0.$$  

The coefficient $A_2$ is the marginal productivity of labor, we assume $A_2 < A_1$.

**Assumption 1** *Sector 2 is represented by a firm.*

There is an additional agent in the model, the *firm* of industry/sector 2. We assume that it is a separate entity, that is, it does not represent any of the other agents of the economy, and we further assume it does not have any kind of political power. There is no possibility that it may influence the political process, it undertakes no lobbying activities, and therefore it plays a neutral role in the elaboration of parties’ platforms and in determining the outcome of elections.

**Assumption 2** *Transfers to group 2 workers may be channelled through sector 2.*

Sector 2 revenues are composed not only by its production but also by transfers $t_{2k}$ from the government/candidate $k$. We depart here from the standard model presented before. Thus, candidates’ platform consists in a transfer $T_{2k}$ made to sector 2 as a whole. The leaky bucket assumption still plays a role, though: From this transfer, only an amount $t_{2k}$ make it to sector 2, and this amount is then splitted between workers and the *firm*. We thus have a “payroll subsidy”. Let us then define sector 2’s gross income as

$$GI_{2k} = Y_2 + N_2 t_{2k} = N_2 (A_2 + t_{2k}),$$
Assumption 3 Income sharing between sector 2 workers and the firm is determined according to the Generalized Nash Bargaining Solution.

We assume the firm has bargaining power $\mu$. The agreement payoffs are given by:

$$\begin{cases}
\text{firm} : GI_{2k} - N_2\omega_{2k} \\
\text{workers} : N_2\omega_{2k}
\end{cases}$$

whereas disagreement payoffs are:

$$\begin{cases}
\text{firm} : 0 \\
\text{workers} : 0
\end{cases}$$

i.e. should no agreement take place both the firm and sector 2 workers get nothing. The Nash Bargaining solution is given by the value of $\omega_{2k}$ that solves the following problem:

$$\max \left\{ (GI_{2k} - N_2\omega_{2k})^\mu (N_2\omega_{2k})^{1-\mu} \right\}$$

The solution is

$$N_2\omega_{2k} = (1 - \mu)N_2(A_2 + t_{2k}) = (1 - \mu)GI_{2k}$$

i.e. a proportion $(1 - \mu)$ of gross income goes to workers, while the remaining $\mu$ to the firm. Note we can still decompose worker’s income as a part of wages $w_2 = (1 - \mu)A_2$ and a part of transfers $(1 - \mu)t_{2k}$. Therefore, the transfer effectively received by workers passes through two filters: the loss accruing between the government and the sector due to the leaky bucket, and the sharing rule applied in sector 2, which depends on the bargaining power of agents.

$$(1 - \mu)t_{2k} = (1 - \mu)(1 - \theta_{2k})T_{2k} \equiv (1 - \tilde{\theta}_{2k})T_{2k}$$

where we introduce the parameter $\tilde{\theta}_{2k} = 1 - (1 - \mu)(1 - \theta_{2k})$, which $\tilde{\theta}_{2k} > \theta_{2k} > 0$ whenever $\mu > 0$. Accordingly we now write –for this case in which both parties tax sector 1 to subsidize sector 2–

$$\tilde{\psi}_k = \frac{1 - \tilde{\theta}_{2k}}{1 + \gamma_{1k}} = \frac{(1 - \mu)(1 - \theta_{2k})}{1 + \gamma_{1k}} = (1 - \mu)\psi_k \quad \text{for} \quad k = A, B.$$
**Assumption 4** Sector 2 can be made operational only after a fixed cost $F$ is invested.

After the bargaining process, the firm has to invest a fixed quantity $F$ in order that the sector can be working. The payoff the firm realizes is a level of profits $\Pi_{2k}$ that writes

$$\Pi_{2k} = \mu G I_{2k} - F = \mu N_2(A_2 + t_{2k}) - F.$$  \hfill (24)

It follows that there exists a level of transfers for which the firm makes zero benefits,

$$\bar{T}_{2k} = \frac{F - \mu N_2 A_2}{(1 - \theta_{2k}) \mu N_2}.$$  \hfill (25)

We now incorporate an extra feature to the model. As equation (25) indicates, the firm’s profits are positive whenever transfers are greater than $\bar{T}_{2k}$. In such a case the analysis is similar to the one introduced in section 2, properly adjusted to introduce the modifications of this section. When transfers are lower than this cutpoint, we are in the “constrained region” and the setting changes.

**Assumption 5** The firm requires non-negative profits to operate Sector 2.

Note that sector 2 profitability heavily depends on government’s transfers. Indeed, according to equation (24), in the absence of transfers firm’s benefits become:

$$\Pi_2 = \mu N_2 A_2 - F,$$

these profits will be negative in the relevant case characterized by $\bar{T}_{2k} > 0$, see equation (25).

An important value of $\chi_k$ we must take into account is the one over which sector 2 is making negative profits. Let us call it $\bar{\chi}_k$, defined as

$$\bar{\chi}_k = \frac{\tilde{\omega}_{1k}}{\tilde{\omega}_{2k}} = \frac{w_1 - (1 + \gamma_{1k})N_2T_{2k}}{w_2 + (1 - \theta_{2k})T_{2k}} = \left(\frac{N_2}{N_1}\right) \frac{\mu (N_1 \psi_k w_1 + N_2 A_2) - F}{(1 - \mu) \psi_k F}$$  \hfill (26)

where the last part of the equation follows from inserting the value of $\bar{T}_{2k}$ given by equation (25) and rearranging. These cutpoints depend negatively on fixed costs $F$.

Note also that $\bar{\chi}_A$ and $\bar{\chi}_B$ are rather similar, the difference comes from the leaky bucket parameters associated to each party. In Figure 5 we show these cutpoints and the regions.
Distribution technologies. Both parties have in principle the possibility to either channelling the transfers to sector 2 workers through the firm or, instead, in a direct way. Direct transfers are those that bypass industry 2, that is they directly reach workers without having an impact on firm’s profits. Under Assumption 5, direct transfers therefore imply sector 2 is shut down, and as a consequence, \( w_2 = 0 \). The total amount available to candidate \( k \) for redistribution (recall equation 22), would fall to \( N_1 \psi_k w_1 \). Indirect transfers, correspondingly, are those that help improve the firm’s profits, thus allowing it to survive, and yielding \( w_2 > 0 \).

We will use the following notation: For any variable \( z \), we use \( \tilde{z} \) if transfers to workers are indirect; \( \hat{z} \) if direct transfers; and \( \bar{z} \) if transfers are such that sector 2 profits are just equal to zero.

We consider hereafter the following case: Industry 2 members are a core support group of party \( A \). This means party \( A \) can more easily get political support from this group using transfers. In terms of the model, the parameter \( \theta_{2A} \) is close to zero. Candidate \( A \) can be therefore thought of as a professional politician who has been
“investing” her time in the sector, and as a result she has a better understanding of their members. A natural consequence of this closeness is party A is ready to intervene should this industry go into problems by contributing to its survival through indirect transfers (which, as seen above, improve the firm’s profitability).

Party A has nevertheless the possibility to let the firm die and make direct transfers. Indeed, the support to sector 2 has a limit: eventually, the total vote it receives falls below the votes it would obtain by proposing direct transfers and letting the firm shut down. When this happens, party A abandons indirect transfers and starts proposing direct ones. However, this has a huge cost in terms of the candidate’s reputation and therefore on its political influence over sector 2 workers. In the event the firm disappears, candidate A would be blamed as responsible. Thus, abandoning indirect transfers means losing the bulk of its ability to get to voters: the parameter $\theta_{2A}$ would increase, resulting in $\hat{\psi}_A$ that falls below $\tilde{\psi}_A$. As shown in the analysis of Figure 2, this implies party A’s best response function moves outwards. The rationale for party A to use indirect transfers is then given by the fact that it is the best it can do taking into account its abilities to redistribute.

In turn, candidate B has no such abilities to get to industry 2 members. Think of him as the outsider, the “reformer” or simply the “new technocrat just arrived from the capital”. He does not have the close relationship with sector 2 workers candidate A has, implying in turn he has no incentives to artificially prolong the survival of industry 2. That is why party B will always propose direct transfers to sector 2 group members. Notice this strategy implies the shut down of this industry.

3.1 Parties best response functions

The best response functions of both parties in the unconstrained region are those of section 2, modified to account for the assumptions we have introduced in this section:

$$\hat{X}_A = \left[ \frac{\delta_1}{\delta_2 \psi_A} \right] \left[ \frac{1}{\chi_B} \right] \left[ 1 \right]$$

(27)
\[ \hat{\chi}_B = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\psi_B} \right]^{\frac{1}{1-\sigma}} \chi_A^{\frac{\sigma}{1-\sigma}} \]  

(28)

It is convenient here to define the level of \( \chi_B \) that produces a best response \( \hat{\chi}_A \) equal to \( \check{\chi}_A(F) \), i.e. where party A transfer to sector 2 is such that firm’s profits are zero: Let \( \chi_{B,0} \) be such that \( \hat{\chi}_A(\chi_{B,0}) = \check{\chi}_A(F) \). Using equation (27) for \( \hat{\chi}_A \) we have

\[ \chi_{B,0} = \left( \frac{\delta_1}{\delta_2} \frac{1}{\psi A} \right)^\frac{1}{\sigma} \left( \frac{1}{\chi_A(F)} \right)^\frac{\sigma}{\sigma-1}. \]

It follows that \( \chi_B > \chi_{B,0} \Leftrightarrow \hat{\chi}_A(\chi_B) < \check{\chi}_A(\chi_B) \) and this implies \( \Pi_{2A} > 0 \). Next, let us define the threshold \( \chi_{B,1} \) below which the total vote of A under direct transfers is greater than the vote under \( \check{T}_{2A} \), i.e. let \( \chi_{B,1} \) be such that \( V_A = \check{V}_A \). Applying this definition, \( \chi_{B,1} \) is implicitly defined by the following equation:

\[ \left( \frac{1}{\chi_{B,1}} \right)^\sigma N_1 \delta_1 [\tilde{\omega}^a_{1A}(\chi_{B,1}) - \tilde{\omega}^a_{1A}] - N_2 \delta_2 [\tilde{\omega}^a_{2A} - \tilde{\omega}^a_{2A}(\chi_{B,1})] = 0. \]

where we make explicit the dependence of \( \omega_{iA} \), the after tax income promised by party A, on party B best response function. Therefore, party A strategy can be summarized by the following best response function:

\[
\chi_A(\chi_B) = \begin{cases} 
\hat{\chi}_A(\chi_B) & \text{if } \chi_B \in (\chi_{B,0}, \infty) \\
\check{\chi}_A(F) & \text{if } \chi_B \in (\chi_{B,1}, \chi_{B,0}) \\
\hat{\chi}_A(\chi_B) & \text{if } \chi_B \in (0, \chi_{B,1}) 
\end{cases}
\]

For levels of \( \chi_B \geq \chi_{B,0} \) party A is on the unrestricted area so the best response function is given by equation (27). For intermediate levels of \( \chi_B \), candidate A best response is \( \check{\chi}_A(F) \) through which it guarantees a level of transfer \( \check{T}_{2A} \) such that sector 2 profits are zero, but sector 2 remains open. Finally, for the lowest values of \( \chi_B \), party A abandons indirect transfers, industry 2 is closed, and direct transfers prevail. Party B, in turn, always proposes direct transfers to group 2 members. Its best response functions is given by equation (28).

Parties’ best response functions are shown in Figure 6. In the case represented there, the equilibrium is found at the intersection of \( \check{\chi}_A(F) \) and \( \hat{\chi}_B(\chi_A) \).
3.1.1 Equilibria

Given the partitioned best response function of party $A$, we may have several equilibria. Moreover, there is a region in which party $A$ has no best response function at all, implying no equilibrium is defined. The precise configuration of an equilibrium depends on parameter levels (such as the ideology distribution and the relative abilities of parties to transfer) and particularly, on the level of fixed costs which positively affects both $\chi_{B,0}$ and $\chi_{B,1}$. We next explore these equilibria, according to the level of $F$.

Given that we want to identify the implemented platform in each configuration, we are interested on finding which candidate wins the elections. For that, we have to feed the model with some particular assumptions on parameters. Let us start out with the following ranking of candidate’s abilities to redistribute: $\hat{\psi}_A > \hat{\psi}_B > \hat{\psi}_A$. Accordingly to what we have said before, party $A$ has the highest ability when using indirect transfers, but if direct transfers prevail it loses reputation vis à vis its electorate, then party $B$ has the highest ability.
Case 1 - low fixed costs. When the level of fixed costs is low enough the thresholds $\chi_{B,0}$ and $\chi_{B,1}$ are also low, so that the equilibrium is most likely found at the intersection of $\tilde{\chi}_A(\chi_B)$ and $\tilde{\chi}_B(\chi_A)$. This is the unconstrained case, for which we adapt the standard model of Section 2, as explained in Subsection 3.1. The intersection of both best response functions yields the equilibrium pair $(\tilde{\chi}_A^*, \tilde{\chi}_B^*)$, with similar characteristics as those of equations (14) and (15). However, since party $B$ proposes direct transfers this implies $w_2 = 0$ and thus the amount available for redistribution is only $\tilde{Y}_B = N_1 \tilde{\psi}_B w_1$. This implies Proposition 4, which identifies the winner of elections in the standard case, fails to hold.

For party $A$ to win it must be the case that $V_A > N/2$. The total vote for party $A$ is given by equation (7) that formulates $V_A$ as a function of thresholds $x_i$. Together with the logarithmic utility function, the assumption on the distribution of ideology, and taking into account the relationship between $x_1^*$ and $x_2^*$ stated by equation (23), candidate $A$ wins the elections if and only if

$$x_2^* > \bar{x}_2$$

where

$$x_2^* = \left( \frac{N_1 \tilde{\psi}_A w_1 + N_2 w_2}{N_1 \tilde{\psi}_B w_1} \right)^{\sigma} = \left( \frac{\tilde{Y}_A}{\tilde{Y}_B} \right)^{\sigma},$$

and

$$\bar{x}_2 = \frac{N_1 \delta_1 \tilde{\psi}_A^\sigma + N_2 \delta_2 \tilde{\psi}_B^\sigma}{N_1 \delta_1 \tilde{\psi}_B^\sigma + N_2 \delta_2 \tilde{\psi}_A^\sigma}.$$

Then, the condition writes

$$V_A > \frac{N}{2} \iff \frac{\tilde{Y}_A}{\tilde{Y}_B} > (\bar{x}_2)^{\frac{1}{\sigma}}.$$

Proposition 5 In the case where the equilibrium is found at the intersection of the best response functions $\tilde{\chi}_A(\chi_B)$ and $\tilde{\chi}_B(\chi_A)$, if $\tilde{\psi}_A > \tilde{\psi}_B$ then candidate $A$ wins the elections.

Proof. Let us analyze how both sides of the inequality behave as functions of $N_2$ and given the leaky bucket parameters embedded into $\psi_k$.

$$\lim_{N_2 \to 0} \left( \frac{\tilde{Y}_A}{\tilde{Y}_B} \right) = \frac{\tilde{\psi}_A}{\tilde{\psi}_B}$$
\[ \frac{\partial \left( \frac{\hat{Y}_A}{Y_B} \right)}{\partial N_2} > 0 \]
\[ \lim_{N_2 \to 0} \left[ \frac{1}{\bar{x}_2^2} \right] = \frac{\hat{\psi}_A}{\psi_B} \]
\[ \frac{\partial \left( \frac{1}{\bar{x}_2^2} \right)}{\partial N_2} < 0 \text{ if } \hat{\psi}_A > \hat{\psi}_B \]
i.e. both sides of the inequality are equal when \( N_2 = 0 \) and, as \( N_2 \) takes positive values, the left hand side grows (for any \( \hat{\psi}_A, \hat{\psi}_B \)) whereas the right hand side decreases if \( \hat{\psi}_A > \hat{\psi}_B \). Therefore, the condition under which \( A \) wins is satisfied for all \( N_2 > 0 \) provided that party \( A \) has more ability to transfer than party \( B \). Note when \( N_2 = 0 \) there is a tie in elections. \( \Box \)

**Case 2 - intermediate fixed costs.** For some level of fixed costs, Party \( A \) enters into the constrained zone, so its best response function is \( \hat{\chi}_A(F) \), through which it guarantees that sector 2 can be open and its workers get their salary. The equilibrium is found at the intersection of this best response function with \( \hat{\chi}_B(\chi_A) \). This is the case represented in Figure 6. In order to find the winner, note this equilibrium may be replicated by using a generic best response function of party \( A \) associated with a particular level of ability to transfer, that we will denote as \( \psi_{fA} \). Recall how these functions are modified when \( \psi_k \) changes, see Figure 2. Therefore, the equilibrium in the constrained zone may be characterized by the intersection of \( \hat{\chi}_B(\chi_A) \) and \( \hat{\chi}_A(\psi_{fA}) \), where \( \psi_{fA} > \hat{\psi}_A \). Therefore, we can still use Proposition 5 and, since \( \psi_{fA} > \hat{\psi}_B \), party \( A \) wins the elections.

**Case 3 - high fixed costs.** Finally, if fixed costs are high, party \( A \) cannot continue to propose indirect transfers and it switches to direct transfers. This has a cost in terms of reputation for this party, that translates into a higher cost of redistribution, modelled through an increase in leaky bucket parameters. Therefore, the equilibrium is found at the intersection of \( \hat{\chi}_A(\chi_B) \) and \( \hat{\chi}_B(\chi_A) \) and it entails the shut down of sector 2 (whoever wins elections). Then, \( w_2 = 0 \) and we have \( Y_k = N_1 \psi_k w_1 \), \( k = \{A, B\} \), so we can still use the results of Proposition 4. Thus, under our assumptions i.e. \( \hat{\psi}_B > \hat{\psi}_A \), party \( B \) wins.
We have shown that, for this particular case of parties abilities we are considering, the inefficient sector is allowed to survive. This is possible because the party which proposes indirect transfers wins the elections, and this is so provided fixed costs are not too high. Note we could interpret these fixed costs in a broader manner, say, as reflecting exogenous variables such as perturbations or other real conditions affecting the economy. Being allowed to do that, the situation of low and intermediate fixed costs would reflect normal economic conditions, whereas a high level would imply bad times. This replicates the experience of many industries and sectors around the world: some of them are born and survive without any kind of external assistance, others, in contrast, need an increasingly amount of support from the government. This may initially count with popular support, given the importance people attach to some industries. Eventually, years of decline and the arrival of new generations may make people and politicians change their minds, and they are shut down. Again, the example of the US automobile industry can be used to illustrate this.

4 Conclusions

The support to a declining industry may have a political rationale. Politicians with a strong electoral support in some areas or sectors may have incentives to prolonge the survival of firms that otherwise would not stand competition. This paper has shown that such a strategy may be a succesful one. When times are good (represented by low fixed costs) and the cost of keeping the inefficient sector alive is low, the candidate who is closer to the sector can easily win elections. That may be of no surprise. However, the paper also shows what happens when times become tougher and the alternative to let the firm die is viewed as increasingly feasible in terms of votes. Eventually, there is no politician who is willing to maintain this artificial situation, so that when fixed costs are high enough both candidates decide to abandon the transfers that help the inefficient sector to survive.

In this sense, this paper shed some light on the debate about the convenience of rescuing and giving financial support to in-trouble firms. Moreover, it can be seen as a
representation of two opposite visions of political intervention in economic activity: on
the one hand, the (perhaps older) approach of parties promising transfers based on purely
electoral basis (machine politics). On the other hand, many contemporaneous politicians
express, at least in their speeches, their commitment to policies that encourage economic
efficiency.

Related to this, we have the traditional debate on two opposite viewpoints: should
governments grant subsidies to inefficient firms as a way to maintain social stability,
employment or other related target? Or should they let those firms die, allowing fac-
tors to relocate to more productive activities while compensating the losers from reform?
5 Appendix

Proof of Proposition 1. We analyse which are the features that an interest group must have in order to be benefitted from redistribution. The problem of, say, party $A$ is the following

$$\max_{T_{iA}} \quad V_A = \sum_{i=1}^{2} N_i \Phi_i(x_i)$$

s.t. \quad $\sum_{i=1}^{2} N_i T_{iA} \leq 0 \quad BC_A$ (29)

The Lagrangean associated to this problem writes:

$$\mathcal{L} = \sum_{i=1}^{2} N_i \Phi_i(x_i) - \lambda_A \sum_{i=1}^{2} N_i T_{iA}$$

(30)

The Lagrange multiplier $\lambda_A$ measures the value to party $A$ of an additional unit available for redistribution, in terms of votes. The first order conditions of this problem are:

$$\frac{\partial \mathcal{L}}{\partial T_{iA}} = N_i \left[ \delta_i \sigma w_i^{\sigma-1} \frac{\partial t_{iA}}{\partial T_{iA}} - \lambda_A^i \right] = 0, \quad i = 1, 2. \quad (31)$$

To see which group is benefitted from redistribution we start out with these first order conditions, evaluating them at $T_{1A} = T_{2A} = 0$, for a given platform $T_{iB}$ of party $B$. The marginal rate of return of an extra unit available for redistribution is given by

$$\lambda_A^1(0) = \delta_1 \sigma w_1^{\sigma-1} \frac{\partial t_{1A}}{\partial T_{1A}}$$

$$\lambda_A^2(0) = \delta_2 \sigma w_2^{\sigma-1} \frac{\partial t_{2A}}{\partial T_{2A}}$$

If $\lambda_A^1(0) > \lambda_A^2(0)$ party $A$ can do better by distributing more to group 1, this means $T_{1A} > 0$. Given the budget constraint, equation (2), this implies $T_{2A} < 0$. Recalling equation (1), leaky bucket parameters are given in this case by $\{\theta_{1A}, \gamma_{2A}\}$ so that

$$\frac{\partial t_{iA}}{\partial T_{iA}} = \begin{cases} (1 - \theta_{1A}) & \text{for } i = 1 \\ (1 + \gamma_{2A}) & \text{for } i = 2 \end{cases} \quad (32)$$

The condition in this case is

$$\lambda_A^1(0) > \lambda_A^2(0) \iff \chi_B < \left[ \frac{w_2}{w_1} \right]^{\frac{1}{\sigma}} \left[ \frac{\delta_1 - 1}{\delta_2} \right]^{\frac{1}{\sigma}} \equiv A1.$$
Inversely, if $\lambda_1^A(0) < \lambda_2^A(0)$ party $A$ will tax group 1 to subsidize group 2. The condition for this is

$$\lambda_1^A(0) < \lambda_2^A(0) \iff \chi_B > \left[ \frac{w_2}{w_1} \right]^{\frac{1+\sigma}{\sigma}} \left[ \frac{\delta_2 1 + \gamma_1^A}{\delta_1 1 - \theta_2^A} \right]^{\frac{1}{\sigma}} \equiv A2.$$ 

Given the fact that leaky bucket parameters lie between 0 and 1, it is the case that $A1 \leq A2$. For given party $B$ platform, group $i$ will benefit from party $A$’s redistribution scheme if its wage $w_i$ is low compared to the other group’s wage, and if it is more dense, i.e. if $\delta_i$ is high. Note there is an intermediate region in which party $A$ does not make any redistribution, provided at least one leaky bucket parameter is different from zero.

The analysis for party $B$ is analogous, yielding the following condition under which party $B$ decides to tax group 2 and to subsidize group 1:

$$\chi_A > \left[ \frac{w_1}{w_2} \right]^{\frac{1+\sigma}{\sigma}} \left[ \frac{\delta_2 1 + \gamma_2^B}{\delta_1 1 + \theta_1^B} \right]^{\frac{1}{\sigma}} \equiv B1.$$ 

And $B$ will tax group 1 to subsidize group 2 whenever:

$$\chi_A < \left[ \frac{w_1}{w_2} \right]^{\frac{1+\sigma}{\sigma}} \left[ \frac{\delta_2 1 - \theta_2^B}{\delta_1 1 + \gamma_1^B} \right]^{\frac{1}{\sigma}} \equiv B2.$$ 

Note that $B1 \geq B2$. The above 4 conditions define the regions we show in Figure 7.

Our equilibrium results of subsection 2.2 must verify the conditions to have $T_{2k} > 0$ and $T_{1k} < 0$, $k = \{A, B\}$, that is $\chi_A^* < B2$ and $\chi_B^* > A2$. Graphically, this equilibrium must be in the northwest quadrant. Then, recalling equation (15), party $A$ taxes group 1 to subsidize group 2 if and only if:

$$\chi_B^* > A2 \iff \frac{w_1}{w_2} > \frac{\delta_1}{\delta_2} \left[ \frac{1}{\psi_A} \right]^{1+\sigma} \psi_B^\sigma.$$ 

Similarly, given equation (14), party $B$ taxes group 1 to subsidize group 2 if and only if:

$$\chi_A^* < B2 \iff \frac{w_1}{w_2} > \frac{\delta_1}{\delta_2} \left[ \frac{1}{\psi_A} \right]^{\sigma} \left[ \frac{1}{\psi_B} \right]^{1-\sigma}.$$ 

We may now perform the same analysis for the remaining 3 quadrants. In the southeast quadrant, both parties tax group 2 to subsidize group 1. First let us find the equilibrium to then check under which conditions it verifies $\chi_A^* > B1$ and $\chi_B^* < A1$. 

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From the first order conditions, equation (31), we find parties’ best response functions:

\[ \chi_A = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\epsilon_A} \right]^{\frac{1}{\sigma}} \left[ \frac{1}{\chi_B} \right]^{\frac{\sigma}{1+\sigma}} \]

\[ \chi_B = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\epsilon_B} \right]^{\frac{1}{\sigma}} \left[ \chi_A \right]^{\frac{\sigma}{1+\sigma}} \]

And the Nash equilibrium:

\[ \chi_A^* = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\epsilon_B} \right]^{\sigma} [\epsilon_A]^{1+\sigma} \]

\[ \chi_B^* = \left[ \frac{\delta_1}{\delta_2} \frac{1}{\epsilon_B} \right]^{1-\sigma} [\epsilon_A]^{\sigma} \]

Finally, it must be the case that this equilibrium verifies the following conditions:

\[ \chi_A^* > B1 \iff \frac{w_1}{w_2} < \frac{\delta_1}{\delta_2} \left[ \frac{1}{\epsilon_B} \right]^{1-\sigma} [\epsilon_A]^{\sigma} \]

\[ \chi_B^* < A1 \iff \frac{w_1}{w_2} < \frac{\delta_1}{\delta_2} \left[ \frac{1}{\epsilon_B} \right]^{\sigma} [\epsilon_A]^{1+\sigma} \]
It remains to be analyzed the southwest and northeast quadrants, in which parties differ in strategies. In the northeast quadrant party A taxes group 1 while party B taxes group 2. The inverse case is found in the southeast quadrant.

The first order conditions, equation (31), for the case $T_{2A} > 0$, $T_{2B} < 0$ (northeast quadrant) yield the following best response functions:

$$
\chi_A = \left[\frac{\delta_1}{\delta_2} \frac{1}{\psi_A}\right]^{\frac{1}{1+\sigma}} \left[\frac{1}{\chi_B}\right]^{\frac{\sigma}{1+\sigma}}
$$

$$
\chi_B = \left[\frac{\delta_1}{\delta_2} \epsilon_B\right]^{\frac{1}{1+\sigma}} \left[\chi_A\right]^{\frac{\sigma}{1+\sigma}}
$$

And the Nash equilibrium:

$$
\chi_A^* = \frac{\delta_1}{\delta_2} \left[\frac{1}{\epsilon_B}\right]^{1+\sigma} \left[\frac{1}{\psi_A}\right]^{1+\sigma}
$$

$$
\chi_B^* = \frac{\delta_1}{\delta_2} \left[\epsilon_B\right]^{1+\sigma} \left[\frac{1}{\psi_A}\right]^{1+\sigma}
$$

This equilibrium must verify

$$
\chi_A^* > B1 \iff \frac{w_1}{w_2} < \frac{\delta_1}{\delta_2} \left[\epsilon_B\right]^{1+\sigma} \left[\frac{1}{\psi_A}\right]^{1+\sigma}
$$

$$
\chi_B^* > A2 \iff \frac{w_1}{w_2} > \frac{\delta_1}{\delta_2} \left[\epsilon_B\right]^{1+\sigma} \left[\frac{1}{\psi_A}\right]^{1+\sigma}
$$

These two regions define a non-empty solution of $\frac{w_1}{w_2}$ only if

$$(1 - \theta_{1B})(1 - \theta_{1A}) > (1 + \gamma_{2B})(1 + \gamma_{2A})$$

and given that every leaky-bucket parameter lies between 0 and 1, this never happens.

Then this case is discarded. Similar results are found for the last case, corresponding to the southwest quadrant, in which party A taxes group 1 and party B taxes group 2. As a result, in the political game with leaky-bucket assumption in which two parties
compete for votes, there are two strategies characterized by the fact that both parties decide to tax the same group.

**Proof of Proposition 4.** We derive the conditions to have party $B$ as the winner (the procedure to show party $A$ wins is analogous). This is so whenever this candidate gets more than half of the votes of the population, $V_B > \frac{N}{2}$, i.e. when

$$N_1 [1 - \Phi_1(x_1^*)] + N_2 [1 - \Phi_2(x_2^*)] > \frac{N}{2}$$

substituting the distribution function given by equation (6) we have

$$\Leftrightarrow N_1 \delta_1 (x_1^* - 1) + N_2 \delta_2 (x_2^* - 1) < 0$$

inserting the relationship between thresholds given by equation (23) we get a condition on $x_2^*$

$$\Leftrightarrow x_2^* < \frac{N_1 \delta_1 \psi_A + N_2 \delta_2 \psi_A}{N_1 \delta_1 \psi_B + N_2 \delta_2 \psi_B}$$

Now, we use the equilibrium value of the cutpoint of $x_2$ given by equation (2.3),

$$\Leftrightarrow \left[ \frac{N_1 \psi_A w_1 + N_2 w_2}{N_1 \psi_B w_1 + N_2 w_2} \right]^{1/\sigma} < \frac{N_1 \delta_1 \psi_A + N_2 \delta_2 \psi_A}{N_1 \delta_1 \psi_B + N_2 \delta_2 \psi_B}$$

Let us denote the right hand side of the above equation by $\bar{x}_2$. Consider the case $\psi_B > \psi_A$. Then,

$$\Leftrightarrow N_1 w_1 \left[ \psi_B (\bar{x}_2)^{1/\sigma} - \psi_A \right] > N_2 w_2 \left[ 1 - (\bar{x}_2)^{1/\sigma} \right]$$

$$\Leftrightarrow \frac{N_1 w_1}{N_2 w_2} > \frac{1 - (\bar{x}_2)^{1/\sigma}}{\psi_B (\bar{x}_2)^{1/\sigma} - \psi_A}.$$ 

Now, recall the condition to have both parties taxing group 1 to subsidize group 2, equation 9. Under $\psi_B > \psi_A$, this is the case if

$$\frac{N_1 w_1}{N_2 w_2} > \frac{N_1 \delta_1 \psi_B^0}{N_2 \delta_2 \psi_A^{1+\sigma}}$$

We next show that when party $B$ has more ability to transfer than party $A$ (i.e. $\psi_B > \psi_A$) and if both parties tax group 1 to subsidize group 2, then party $B$ wins the elections. This is so whenever:

$$\frac{N_1 \delta_1 \psi_B^0}{N_2 \delta_2 \psi_A^{1+\sigma}} > \frac{1 - (\bar{x}_2)^{1/\sigma}}{\psi_B (\bar{x}_2)^{1/\sigma} - \psi_A}$$
\[(\bar{x}_2)^\frac{1}{\sigma} > \frac{\psi_A (N_1 \delta_1 \psi_B + N_2 \delta_2 \psi_A)}{N_1 \delta_1 \psi_B^{1+\sigma} + N_2 \delta_2 \psi_A^{1+\sigma}}\]

Inserting the value of \((\bar{x}_2)\) and rearranging,

\[\Leftrightarrow N_1 \delta_1 + N_2 \delta_2 > \left(\frac{N_1 \delta_1 \psi_B + N_2 \delta_2 \psi_A}{(N_1 \delta_1 \psi_B^{1+\sigma} + N_2 \delta_2 \psi_A^{1+\sigma})^{\sigma}}\right)^{1+\sigma}\]

Let us call the right hand side of this condition \(\Gamma(\psi_B, \psi_A, \delta_i, \sigma, N_i)\). Note

(i). \(\Gamma(0, \psi_A, \delta_i, \sigma, N_i) = N_2 \delta_2\)

(ii). \(\frac{\partial \Gamma}{\partial \psi_B} < 0\) if \(\psi_B > \psi_A\).

Then, the condition holds if \(\psi_B > \psi_A\). Therefore, in the case in which both parties tax group 1 to subsidize group 2, if \(\psi_B > \psi_A\) then party B wins the elections. Under \(\psi_A > \psi_B\), party A wins the elections. \(\Box\)
References


